

Calculations In Chemistry



Modules 17 and 18

Ideal Gases, Gas Labs, Gas Reactions

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Module 17 — Ideal Gases

Timing: Ideal gas law calculations are covered in Module 17. The laws discovered by Boyle, Charles, Avogadro, and Dalton are addressed in Module 18. Kinetic Molecular Theory and Graham's law calculations are covered in Module 19. If you need to solve calculations for Module 18 or 19 topics before those using the ideal gas law, complete Lessons 17A and 17D, then Modules 18 and/or 19.

Prerequisites: It will *help* with the Module 17 calculations if you have completed Modules 2, 4, 5, 8 and Lesson 11B. Those lessons can be done quickly.

Pretest: If you believe you have previously mastered ideal gas law calculations, try the problems in the last lesson in this module. If you can do those, you may skip this module.

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Lesson 17A: Gas Measurements and Fundamentals

Gas Quantities and Their Units

Chemistry is most often concerned with matter in 3 states: gas, liquid, and solid. The gas state is in most respects the easiest to study, because by most measures, gases have similar and highly predictable behavior. Gas quantities can be measured using **4** variables:

- Pressure, volume, temperature, and moles of gas particles.

The symbols for these variables are **P**, **V**, **T**, and **n**.

Gas Pressure

In an experiment, a glass tube about 100 cm (1 meter) long is sealed at one end and then filled with mercury: an element that is a dense, silver-colored metal (symbol **Hg**) and is a *liquid* at room temperature. The open end of the filled tube is covered, the tube is turned over, and the covered end is placed under the surface of additional mercury in a partially filled beaker. The tube end that is under the mercury in the beaker is then uncovered.

What happens? In all experiments at standard atmospheric pressure, the same result. The top of the mercury in the tube quickly falls from the top of the tube until it is *about* 76 centimeters above the surface of the mercury in the beaker. There, the mercury descent stops. The result is a column of mercury inside the glass tube that is about 76 cm high.

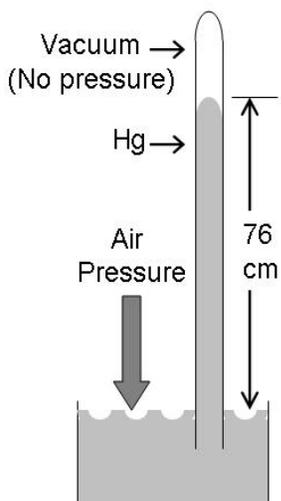
What is in the tube *above* the mercury column? A bit of mercury vapor, but no air. The space above the mercury in the tube is mostly empty: close to a **vacuum**.

What happens if the top of the tube is snapped off? The mercury inside the tube behaves the same as a straw full of liquid when you take your finger off the top. The liquid mercury in the tube falls quickly until it reaches the same level as the mercury in the beaker.

However, as long as the tube is sealed and is longer than 76 cm, the top of the mercury in the tube will remain about 76 cm above the top of the mercury in the beaker.

Why? The device made in this experiment is a mercury **barometer**. It measures the **pressure** of the air outside the tube.

A barometer is a kind of balance, like a playground “teeter-totter.” The pressure of the dense column of mercury *in* the tube, pressing down on the top of the pool of mercury in the beaker, is *balanced* by the pressure of the 20-mile-high column of air, the atmospheric pressure, pressing down on the pool of Hg *outside* the tube.



We could construct our barometer using water as the liquid. However, for water to balance the air, since water is about 13.5 times less dense than mercury, our tube would need to be about 13.5 times higher, about 33 feet high, roughly 3-4 stories on a typical building.

If the air pressure outside the mercury column increases, the mercury is pushed higher in the tube. If the atmospheric pressure is lowered, the mercury level in the tube falls.

When a weather forecast states that “the barometric pressure is 30.04 inches and falling,” it is describing the height of the mercury column in a barometer (76 cm is about 30 inches). In meteorology, a rising barometer, or a high pressure system, is usually associated with fair weather. Falling barometers and low pressure systems are often associated with clouds, storms, and precipitation.

Measuring Pressure

In the branches of science, pressure is measured in a variety of units.

Chemistry defines **standard pressure** as a gas pressure of exactly 760 mm (76 cm) of mercury as measured in a barometer. This is also known as exactly **one atmosphere** of pressure. Normal atmospheric pressure at sea level on a fair-weather day is *about* one atmosphere.

The SI base units for pressure are termed **pascals** (abbreviated Pa) in honor of the 17th Century French mathematician and scientist Blaise Pascal, whose experiments with gases led him to propose the concept of vacuum. (Pascal’s contemporary, the scientist and mathematician René Descartes, disagreed with the vacuum concept, writing that Pascal had “too much vacuum in his head.”)

The following table of pressure units must be memorized. These equalities will be used to convert among pressure units.

<u>Pressure Units</u>	
Standard Pressure	\equiv 1 atmosphere (\equiv means “is defined as equal to”)
	\equiv 760 mm Hg (mercury) \equiv 760 torr
	= 101 kilopascals (kPa) (not exact; not a definition)
	= 1.01 bars

Any two of those measures can be used in a *conversion factor* for pressure units.

Examples: 1 atm = 760 mm Hg or 101 kPa = 760 torr

Practice A

Practice until you can write the table of pressure unit definitions from memory, then use the definitions as conversions in these calculations. Answers are at the end of this lesson.

1. The lowest atmospheric pressure at sea level was recorded in 1979 during Typhoon Tip; a pressure of 870. millibars. What is this pressure in kPa?
 2. If 2.54 cm \equiv one inch (exactly), and standard pressure is defined as exactly 760 mm Hg, what is standard pressure in inches of mercury?
 3. Standard pressure in English units is 14.7 pounds per square inch (psi). If a bicycle tire has a pressure of 72 psi, how many atmospheres would this be?
 4. How many kPa is 25.0 torr?
-

Gas Volumes

Gas volumes are measured in standard metric volume units: Liters (dm³) and mL (cm³).

Gas Temperature

Temperature is defined as the **average kinetic energy** of particles.

Kinetic energy is energy of **motion**, calculated by the equation

$$\text{Energy of Motion} = 1/2 (\text{mass}) (\text{velocity})^2, \text{ or } \boxed{\text{KE} = 1/2 mv^2}$$

Since the chemical particles of a substance have a constant mass, this equation means that when molecules move *twice* as fast, they must have “2 squared,” or *four* times as much kinetic energy, and their **absolute temperature** must be four times higher.

One of the implications of this equation is that, though particles cannot have zero mass, they *can* have zero velocity: they can (in theory) stop moving.

If all of the molecules in a sample had zero velocity, the temperature of the sample would be $-273.15\text{ }^{\circ}\text{C}$, which is defined as **absolute zero**. Absolute zero is the *bottom* of the temperature scale. Nothing can be colder than absolute zero.

The **Kelvin** (or **absolute**) temperature scale simplifies the mathematics of calculations based on gas temperatures. The Celsius scale defines **0** degrees as the melting and freezing temperature of water, and 100 degrees as the boiling temperature of water at standard pressure. The Kelvin scale keeps the *same size degree* as Celsius, but defines **0** as absolute zero.

The equation relating the Kelvin and Celsius scales is $\boxed{\text{K} = ^{\circ}\text{C} + 273}$ (use 273.15 in calculations if additional precision is needed). This equation must be memorized. The SI unit that measures absolute temperature is termed the **kelvin**. A temperature in kelvins is always 273 degrees *higher* than the temperature in degrees Celsius.

Temperature measurements must be converted to kelvins when using any equation which specifies a capital T.

Practice B

	<u>In kelvins</u>	<u>In °Celsius</u>
1. Absolute Zero	0 K	−273 °C
Water Boils	373 K	100. °C
Nitrogen Boils	77 K	−196 °C
Table Salt Melts	<u>1074 K</u>	801 °C
Water Freezes	273 K	0 °C
Std. Temperature	273 K	0 °C

Note that when using $K = ^\circ C + 273$ you are adding or subtracting, and the significant figures are determined by the highest *place* with doubt. All of the numbers above have doubt in the one's place.

2. In a problem involving gases, you calculate a temperature for the gas of -310 degrees Celsius. Your answer is.....? Mistaken. $-310\text{ }^\circ\text{C}$ is below absolute zero ($-273\text{ }^\circ\text{C}$). There cannot be a temperature colder than absolute zero.

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Lesson 17B: Gases at STP

Prerequisites: You should be able to do the calculations in this lesson if you have completed Modules 2, 4, 5, and 8, plus Lessons 11B and 17A.

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Standard Temperature and Pressure (STP)

In experiments with gases, it is preferred to measure gas *volumes* at **standard temperature and pressure**, abbreviated **STP**. The following values must be committed to memory.

Standard Temperature $\equiv 0^\circ\text{C} = 273.15\text{ K}$ = the temperature of an ice-water mixture
 Standard Pressure $\equiv 1\text{ atmosphere} \equiv 760\text{ mm Hg} \equiv 760\text{ torr}$
 $= 101\text{ kilopascals (kPa)} = 1.01\text{ bars}$

Molar Gas Volume At STP

Gases have remarkably consistent behavior. One important example involves gas volumes: if two samples of gas have the same number of gas molecules and they are at the same temperature and pressure, the two samples will (in most cases) occupy the same volume. This rule is true under most conditions even if the gases have different molar masses or different molecular formulas. It is even true for different mixtures of gases.

In addition, if we know the temperature, pressure, and number of moles of gas particles in a sample, the gas volume will be predictable.

For example: In *most* cases, for one mole of any gas or any mixture of gas molecules, the volume at STP will be 22.4 liters.

The volume of a mole of gas at STP (the molar volume) provides us with a conversion factor for gas calculations at STP.

Our rule will be

The STP Prompt

In calculations, if a gas is stated to be *at STP* or is at a T and P that are equivalent to STP, write in the DATA:

$$\mathbf{1 \text{ mole gas} = 22.4 \text{ L gas at STP}}$$

For gases at STP, we will use the STP prompt to solve calculations by using conversions.

Attaching P and T to V

Note that in this equality, “at STP” is attached only to the gas *volume*. Because gas volumes vary with temperature and pressure, *all* gas volumes *must* have pressure and temperature conditions stated if the volume is to be a measure of the number of particles in the sample.

The rule is: Gas *volumes* must be labeled with a T and P if the T and P are known.

In calculations, when you write gas volumes (with units such as L or mL), attach a P and T.

Non-Ideal Behavior

When gases approach a pressure and temperature at which they condense (become liquids or solids), the *ideal* gas assumptions of the STP prompt begin to lose their validity. At pressures above standard pressure, variations from *ideal* behavior can also become substantial. However, unless a problem indicates non-ideal gas behavior, if the conditions are at STP, you should assume that the STP mole to volume relationship applies.

Practice A

1. Write values for standard pressure using 5 different units.
2. Write values for standard temperature in 2 different units.

Calculations For Gases At STP

The STP prompt (relating gas liters to moles) is often used in conjunction with

- The grams prompt (write the grams to moles relationship), and
- The Avogadro prompt (write the particles to moles relationship).

The purpose of the prompts is to get into your DATA table the conversions that will allow you to convert between the units in the problem and *moles*. In chemistry, most fundamental relationships are defined as ratios that involve moles. Having the relationships in the DATA table that relate the units to moles will allow you minimize interruptions while applying conversions to solve.

Together, these prompts allow you to solve most STP gas calculations using conversions. You will also need to recall that if a problem asks for

- molar mass, you WANT grams *per* 1 mole;
- density, you WANT a mass unit (g or kg) over 1 volume unit (L, dm³, mL).

Try this problem in your notebook, then check your answer below.

Q1. 2.0×10^{23} molecules of NO₂ gas would occupy how many liters at STP?

* * * * *

Answer

WANT: ? L NO₂ gas at STP = (first, write the unit you WANT)

DATA: 2.0×10^{23} molecules of NO₂ gas

6.02×10^{23} of anything = 1 mol anything (Avogadro Prompt)

1 mol any gas = 22.4 L any gas at STP (STP Prompt)

Strategy: Want a single unit? Start with a single unit, and find moles first.

SOLVE:

$$\begin{aligned} ? \text{ L NO}_2(\text{g}) \text{ STP} &= 2.0 \times 10^{23} \text{-molec. NO}_2 \bullet \frac{1 \text{ mol}}{6.02 \times 10^{23} \text{ molec.}} \bullet \frac{22.4 \text{ L gas STP}}{1 \text{ mol gas}} = \\ &= 7.4 \text{ L NO}_2(\text{g}) \text{ at STP} \end{aligned}$$

Note the difference between these calculations and stoichiometry. The above problem involved only *one* substance. If stoichiometry steps are needed, you will see DATA for *two* substances involved in a chemical *reaction*, and you will need a mol/mol conversion when you solve.

Working the Examples

An effective technique in learning to solve problems in the physical sciences (and math) is to *work the examples* in the textbook. To do so: *cover* the answer, read the example, and try to solve. If you need help, peek at the answer, and then try the example again.

Try that method, this time solving for a ratio unit.

Q2. Determine the density of O₂ gas at STP in grams per milliliter.

HINT: When solving for a *ratio*, if a calculation involves only *one* substance, the rule “want a ratio, start with a ratio” will usually solve faster than “solve for the top and bottom units separately.”

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Answer

WANTED: $\frac{? \text{ g O}_2 \text{ gas}}{\text{mL at STP}}$

DATA: $32.0 \text{ g O}_2 = 1 \text{ mol O}_2$ (grams prompt)
 $1 \text{ mol any gas} = 22.4 \text{ L any gas at STP}$ (STP Prompt)

SOLVE: (Want a ratio? Start with a ratio. See Lesson 11B.)

$$\frac{? \text{ g O}_2 \text{ gas}}{\text{mL gas STP}} = \frac{32.0 \text{ g O}_2}{1 \text{ mol O}_2} \cdot \frac{1 \text{ mol gas}}{22.4 \text{ L gas STP}} \cdot \frac{10^{-3} \text{ L}}{1 \text{ mL}} = \frac{0.00143 \text{ g O}_2}{\text{mL at STP}}$$

Your answer may have those 3 conversions, right-side up, in any order. Note that STP was attached to the gas *volume* unit (mL).

Note also that

- The conversions for *molar mass* and *particles per mole* are valid whether the substance is a gas, liquid, or solid.
- The *STP* prompt, however, only works for gases, and only works at STP.

Practice B

Make certain that you can write all 3 of the above prompts from memory. Then do **all** of Problems 1 below. After #1, do every other problem, and more if you need more practice.

- Write the *units* WANTED when you are asked to find
 - Molarity
 - Molar Mass
 - Volume
 - Mass
 - Density
 - Speed or Velocity
 - Gas Pressure
 - Temperature
- Calculate the density of SO₂ gas in g/L at STP.
- The density of a gas at STP is 0.00205 g•mL⁻¹. What is its molar mass?
- If 250. mL of a gas at STP weighs 0.313 grams, what is the molar mass of the gas?
- Calculate the number of molecules in 1.12 L of CO₂ gas at STP.
- Calculate the volume in mL of 15.2 grams of F₂ gas at 273 K and standard pressure.
- If 0.0700 moles of a gas has a volume of 1,760 mL, what is the volume of one mole of the gas in liters under the same temperature and pressure conditions?
- Calculate the density in kg•L⁻¹ of radon gas at STP.

ANSWERS**Practice A**

- Standard P = 1 Atmosphere \equiv 760 mm Hg \equiv 760 Torr = 101 kilopascals (kPa) = 1.01 bars
- Standard T = **0 °C** or **273 K**

Practice B

- Molarity $\frac{\text{Moles}}{1 \text{ L solution}}$
 - Molar Mass $\frac{\text{grams}}{1 \text{ mole}}$
 - Volume **L, mL, dm³, cm³**
 - Mass **kg, g, ng**
 - Density $\frac{\text{any mass unit (kg or g)}}{\text{any volume unit (L, mL, dm}^3\text{)}}$
 - Speed $\frac{\text{any distance (cm, miles)}}{\text{any time (sec, hour)}}$
 - Gas Pressure **atm, torr, mm Hg, kPa, bars**
 - Temperature **°C or K**
- WANTED: $? \frac{\text{g SO}_2 \text{ gas}}{\text{L SO}_2 \text{ gas at STP}}$ (Write *ratio* units WANTED as *fractions*)

DATA: 64.1 g SO₂ = 1 mol SO₂ (grams prompt)
1 mol any gas = 22.4 L any gas at STP (STP Prompt)

SOLVE: (Want a ratio? Start with a ratio. Since grams is on top in the answer, you could start with grams on top as the given ratio, but these conversions can be in any order.)

$$? \frac{\text{g SO}_2 \text{ gas}}{\text{L SO}_2 \text{ gas at STP}} = \frac{64.1 \text{ g SO}_2}{1 \text{ mol SO}_2} \bullet \frac{1 \text{ mol gas}}{22.4 \text{ L gas STP}} = \frac{2.86 \text{ g SO}_2(\text{g})}{\text{L SO}_2 \text{ gas at STP}}$$
- WANT: $? \frac{\text{g}}{\text{mol}}$ (write the *unit* WANTED for molar mass: g/mol)

DATA: 0.00205 g gas = 1 mL gas at STP ($\text{g} \cdot \text{mL}^{-1} = \text{g} / \text{mL}$)
1 mol any gas = 22.4 L any gas at STP (STP Prompt)
(Note that the grams prompt only works if you know a substance *formula*.)

SOLVE: (the conversions below may be in any order, so long as they are right-side up.)

$$? \frac{\text{g}}{\text{mol}} = \frac{0.00205 \text{ g gas}}{1 \text{ mL gas at STP}} \bullet \frac{1 \text{ mL}}{10^{-3} \text{ L}} \bullet \frac{22.4 \text{ L any gas at STP}}{1 \text{ mol gas}} = \frac{45.9 \text{ g}}{\text{mol}}$$
- WANT: $? \frac{\text{g}}{\text{mol}}$

DATA: 0.313 g gas = 250. mL gas at STP
1 mol any gas = 22.4 L any gas at STP (STP Prompt)

SOLVE: (These conversions below may be in any order.)

$$? \frac{\text{g}}{\text{mol}} = \frac{0.313 \text{ g gas}}{250. \text{ mL gas at STP}} \bullet \frac{1 \text{ mL}}{10^{-3} \text{ L}} \bullet \frac{22.4 \text{ L any gas STP}}{1 \text{ mol gas}} = 28.0 \frac{\text{g}}{\text{mol}}$$

Reminders

- Attach temperature and pressure conditions, if known, to gas *volumes*.
- In the interest of readability, most unit cancellations in these answers are left for you to do. However, in your work, always ~~mark~~ your unit cancellations as a check on your conversions.

5. WANT: ? molecules CO₂ gas =

DATA: 1.12 L CO₂ gas at STP

6.02 x 10²³ anything = 1 mol anything (Avogadro Prompt)

1 mol any gas = 22.4 L any gas at STP (STP Prompt)

SOLVE:

$$? \text{ molecules CO}_2(g) = 1.12 \text{ L CO}_2(g) \text{ STP} \bullet \frac{1 \text{ mol gas}}{22.4 \text{ L gas STP}} \bullet \frac{6.02 \times 10^{23} \text{ molec.}}{1 \text{ mole}} = 3.01 \times 10^{22} \text{ molec. CO}_2$$

6. WANT: ? mL F₂ gas at STP = (these conditions are STP)

DATA: 15.2 g F₂ gas (single unit *given*)

38.0 g F₂ gas = 1 mol F₂ gas

1 mol any gas = 22.4 L any gas at STP

SOLVE: (Want a single unit?)

$$? \text{ mL F}_2(g) \text{ STP} = 15.2 \text{ g F}_2(g) \bullet \frac{1 \text{ mol F}_2}{38.0 \text{ g F}_2} \bullet \frac{22.4 \text{ L gas STP}}{1 \text{ mol gas}} \bullet \frac{1 \text{ mL}}{10^{-3} \text{ L}} = 8.96 \times 10^3 \text{ mL F}_2(g) \text{ STP}$$

7. WANTED: ? L gas at given P and T

1 mol

(Strategy: If the problem is asking for a unit per one unit, it is asking for a ratio unit.

This problem does not specify STP, but you can solve for the requested unit.

Compare units: You WANT *liters* and *moles*. You are *given mL* and *moles*.)

DATA: 0.0700 mol gas = 1,760. mL gas at given T and P

SOLVE: (Since answer unit *moles* is on bottom, and answer unit *liters* is not in the data, you might start with moles on the bottom in your *given* ratio.)

$$? \frac{\text{L gas at given T and P}}{1 \text{ mole}} = \frac{1,760 \text{ mL gas at given T and P}}{0.0700 \text{ mole}} \bullet \frac{10^{-3} \text{ L}}{1 \text{ mL}} = \frac{25.1 \text{ L gas at given T and P}}{\text{mole}}$$

8. Hint: The grams prompt applies to kilograms, too. If needed, adjust your work and try again.

* * * * *

WANTED: $\frac{? \text{ kg Rn gas}}{\text{L at STP}}$ ($\text{kg} \cdot \text{L}^{-1} = \text{kg} / \text{L}$. Write ratio units WANTED as *fractions*)

DATA: 222 g Rn = 1 mol Rn ($\text{kg} = \text{g}$ prompt)
 1 mol any gas = 22.4 L any gas at STP (STP Prompt)

SOLVE: (Start with a ratio. In your *given*, you may want one unit where it belongs in the answer, but your conversions may be in any order that cancels to give the WANTED unit.)

$$? \frac{\text{kg Rn gas}}{\text{L Rn gas at STP}} = \frac{1 \text{ mol gas}}{22.4 \text{ L gas STP}} \bullet \frac{222 \text{ g Rn}}{1 \text{ mol Rn}} \bullet \frac{1 \text{ kg}}{10^3 \text{ g}} = 9.91 \times 10^{-3} \frac{\text{kg Rn}}{\text{L Rn gas at STP}}$$

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Lesson 17C: Cancellation of Complex Units

Solving gas law calculations requires working with **complex units**: those with *reciprocal* units or *more* than one unit in the numerator or denominator. Let's review the mathematics of working with complex fractions.

Why Not To Write "A/B/C"

When solving complex fractions, the notation for the fractions must be handled with care. Why? Try these problems.

A. $\frac{8}{4}$ divided by 2 = B. 8 divided by $\frac{4}{2}$ = C. $8/4/2$ =

* * * * *

Answer

- A. 2 divided by 2 = 1 B. 8 divided by 2 = 4
 C. Could be 1 or 4, depending on which is the fraction: 8/4 or 4/2.

The format A/B/C for numbers or units is ambiguous unless you know, from the context or from prior steps, which is the fraction.

Let's try problem C again, this time with the fraction identified by parentheses. Use the rule of algebra: perform the operation inside the parentheses *first*.

D. $(8/4)/2 =$ E. $8/(4/2) =$

* * * * *

D. $2/2 = 1$ E. $8/2 = 4$

With the parentheses, the problem is easy. But in part C, for $8/4/2$ without the parentheses, you cannot be sure of the right answer. For this reason, writing numbers or units in a format $A/B/C$ should be avoided.

In these lessons, we will either use a *thick underline*, or group fractions in parentheses, to distinguish a numerator from a denominator.

Multiplying and Dividing Fractions

By definition: 1 divided by $(1/X)$ = the **reciprocal** of $1/X$ = $\frac{1}{\frac{1}{X}} = X$

Recall that $1/X$ can be written as X^{-1} . Another way to state the relationships above is to apply the rule: when you take an exponential to a power, you multiply the exponents:

$$(X^{-1})^{-1} = (X^{+1}) = X.$$

The rule $\frac{1}{(1/X)} = X$ is an example of this general rule:

Complex fraction Rule 1: To simplify the *reciprocal* of a *fraction*, invert the fraction.

Apply Rule 1 to the following problem, then check your answer below.

Q1. Remove the parentheses and simplify: $1/(B/C) =$

★ ★ ★ ★ ★

Answer: The reciprocal of a fraction simplifies by inverting the fraction.

$1/(B/C)$ simplifies by inversion (flips the fraction over) to C/B .

In symbols: $\frac{1}{\frac{B}{C}} = \frac{C}{B}$ In exponents: $(B/C)^{-1} = (B \cdot C^{-1})^{-1} = B^{-1} \cdot C$

Calculations in chemistry may involve fractions in the numerator, denominator, or both. To handle these cases systematically, use the following rule.

Complex fraction Rule 2: When a term has *two* fraction lines (either _____ or /), *separate* the terms that are *fractions*. To do so, apply these steps in this order.

- a. If a term has a fraction in the *denominator*, separate the terms into ($1/\text{fraction in the denominator}$) multiplied by the remaining terms.
- b. If there is a fraction in the *numerator*, *separate* and multiply that fraction by the other terms in the numerator or denominator.
- c. Then simplify: *invert* any reciprocal *fractions*, cancel units that cancel, and multiply the terms.

Let's learn these rules using examples. Solve Q2 to Q4 below in your notebook.

Q2. Remove the parentheses and simplify: $A/(B/C)$ Apply Rule 2a, then 2c.

* * * * *

$$\text{Answer: } A/(B/C) = \frac{A}{\frac{B}{C}} = A \cdot \frac{1}{\frac{B}{C}} = A \cdot \frac{C}{B} = \boxed{\frac{A \cdot C}{B}}$$

↑Rule 2a ↑Rule 2c.

Since there is a fraction in the denominator, separate that fraction into a reciprocal times the other terms, then invert the fraction and multiply.

You can also solve using exponents: $A \cdot (B \cdot C^{-1})^{-1} = A \cdot B^{-1} \cdot C$

Q3. Simplify $(A/B)/C$ Apply Rule 2b.

* * * * *

Answer: Note these three different but equivalent ways of representing this problem, and then how the *answer* differs from the previous example.

$$(A/B)/C = \frac{\frac{A}{B}}{C} = \frac{A}{B} \cdot \frac{1}{C} = \boxed{\frac{A}{B \cdot C}}$$

Rewrite for clarity[^] ^Rule 2b

Since there is a fraction in the numerator, separate that fraction from other terms.

A way to summarize Rule 2 is:

To simplify a term with more than one fraction, separate the fractions.

Q4. Simplify $(A/B)/(C \cdot (D/E)) =$

* * * * *

$$(A/B)/(C \cdot (D/E)) = \frac{\frac{A}{B}}{C \cdot \frac{D}{E}} = \frac{A}{B} \cdot \frac{1}{\frac{C \cdot D}{E}} = \frac{A}{B} \cdot \frac{1}{C} \cdot \frac{1}{\frac{D}{E}} = \boxed{\frac{A \cdot E}{B \cdot C \cdot D}}$$

Rewrite for clarity[↑] ↑Rule 2a ↑Rule 2b ↑Rule 2c

In solving problems, when *you* are *writing* terms with two or more fractions, you will need to develop a systematic way to distinguish fractions in the numerator and denominator in cases where confusion may occur.

Practice A: Learn the rules, then simplify these.

1. $X/(Y/Z) =$

2. $(D/E)^{-1} =$

$$3. \frac{\text{meters}}{\frac{\text{meters}}{\text{second}}} =$$

$$4. \frac{(\text{meters/second})}{\text{second}} =$$

$$5. \text{meters/second/second} =$$

Dividing Complex Numbers and Units

For some derived quantities and constants, units are complex fractions. Examples that we will soon encounter include

$$\text{The Gas Constant} = R = 0.0821 \frac{\text{atm} \cdot \text{L}}{\text{mole} \cdot \text{K}}$$

$$\text{The specific heat capacity of water} = c_{\text{water}} = 4.184 \text{ joule/gram} \cdot \text{K}$$

Joule (J) is a unit that measures energy. In those units, the *dot* between two units means that the two units are multiplied together in either the numerator or denominator. In 4.184 joule/**gram**•K, grams and kelvins are both in the *denominator*.

$$4.184 \text{ joule/gram} \cdot \text{K} = 4.184 \text{ joule/}(\text{gram} \cdot \text{K}) = 4.184 \text{ J} \cdot \text{g}^{-1} \cdot \text{K}^{-1}$$

(If your course at this point uses the *unit*⁻¹ notation frequently in calculations, you should complete Lesson 27F after this lesson.)

When multiplying and dividing terms with complex units, you must do the math for both the numbers and the units. When units are found in *fractions*, unit cancellation follows the rules of algebra reviewed in the section above. Try this problem.

Q. Solve for the number and the unit of the answer.

$$\frac{360 \text{ joules}}{18.0 \text{ K} \cdot 0.50 \frac{\text{joules}}{\text{gram} \cdot \text{K}}} =$$

* * * * *

Answer

Note how the units are rearranged in each step.

$$\frac{360 \text{ joules}}{18.0 \text{ K} \cdot 0.50 \frac{\text{joules}}{\text{gram} \cdot \text{K}}} = \frac{360 \text{ joules}}{18.0 \text{ K}} \cdot \frac{1}{0.50 \frac{\text{joules}}{\text{gram} \cdot \text{K}}} = \frac{360 \text{ joules}}{18.0 \text{ K}} \cdot \frac{\text{gram} \cdot \text{K}}{0.50 \text{ joules}} = 40. \text{ g}$$

Solving uses Rule 2a: if there is a unit that is a fraction in the denominator, *separate* the fraction into **1/fraction in the denominator** multiplied by the other terms. Then, invert the reciprocal, cancel units that cancel, and multiply.

Mark the unit cancellation in the final step above.

Cancellation Shortcuts

When canceling numbers and units in complex fractions, you can often simplify by first canceling separately *within* the numerator and denominator, and then canceling *between* the numerator and denominator. However, when using this shortcut, you must remember that canceling a number or unit does *not* get rid of it: it replaces it with a 1. Try this example.

Q. First cancel units in the denominator, then between the numerator and denominator:
$$\frac{360 \text{ joules}}{12 \text{ K} \cdot 0.20 \frac{\text{joules}}{\text{gram} \cdot \text{K}}} =$$

* * * * *

Answer

$$\frac{360 \text{ joules}}{12 \text{ K} \cdot 0.20 \frac{\text{joules}}{\text{gram} \cdot \text{K}}} = \frac{360 \cancel{\text{joules}}}{12 \text{ K} \cdot 0.20 \cancel{\frac{\text{joules}}{\text{gram} \cdot \text{K}}}} = \frac{360}{12 \cdot 0.20} \cdot \frac{1}{\frac{1}{\text{gram}}} = 150 \text{ g}$$

In the third term, the 1's are important. $1/\text{grams}$ and $1/(1/\text{grams})$ are not the same.

When in doubt about how to apply this type of unit cancellation, skip the shortcuts and use systematic Rules 1-2c.

Practice B: Apply the rules above to simplify these.

- $\frac{\text{calories}}{\text{calorie} \cdot \text{gram}} \cdot \text{gram} =$
- $\frac{\text{atm} \cdot \text{L}}{(\text{mole}) \cdot \frac{\text{atm} \cdot \text{L}}{\text{mole} \cdot \text{K}}} =$
- $\frac{(\text{mol}) \cdot \frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}} \cdot (\text{K})}{\text{liters}} =$

ANSWERS**Practice A**

- $X/(Y/Z) = X \cdot \frac{1}{\frac{Y}{Z}} = \frac{X \cdot Z}{Y}$
- $(D/E)^{-1} = \frac{1}{\frac{D}{E}} = \frac{E}{D}$ or $(D^{-1} \cdot E)$
- $\frac{\text{meters}}{\text{second}} = \text{meters} \cdot \frac{1}{\frac{\text{meters}}{\text{second}}} = \text{meters} \cdot \frac{\text{second}}{\text{meters}} = \text{seconds}$
- $\frac{(\text{meters}/\text{sec})}{\text{second}} = \frac{\text{meters}}{\text{second}} \cdot \frac{1}{\text{second}} = \frac{\text{meters}}{\text{second}^2}$
- $\text{meters}/\text{second}/\text{second} =$ Cannot be evaluated unless you know the numerator or denominator.

Practice B

$$1. \frac{\text{calories}}{\frac{\text{calorie}}{\text{gram}} \cdot \text{gram}} = \frac{1}{\frac{\text{calorie}}{\text{gram}} \cdot \text{gram}} \cdot \frac{\text{calories}}{\text{gram}} = \frac{\text{gram} \cdot \text{°C}}{\text{calorie}} \cdot \frac{\text{calories}}{\text{gram}} = \text{°C}$$

$$2. \frac{\text{atm} \cdot \text{L}}{(\text{mole}) \cdot \frac{\text{atm} \cdot \text{L}}{\text{mole} \cdot \text{K}}} = \frac{\text{atm} \cdot \text{L}}{\text{mole}} \cdot \frac{1}{\frac{\text{atm} \cdot \text{L}}{\text{mole} \cdot \text{K}}} = \frac{\text{atm} \cdot \text{L}}{\text{mole}} \cdot \frac{\text{mole} \cdot \text{K}}{\text{atm} \cdot \text{L}} = \text{K}$$

$$3. \frac{(\text{mol}) \cdot \frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}}}{\text{liters}} \cdot (\text{K}) = (\text{mol}) \cdot \left(\frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}} \right) \cdot (\text{K}) \cdot \frac{1}{\text{L}} = \text{atm}$$

* * * * *

Lesson 17D: The Ideal-Gas Law and Solving Equations**Ideal Gases**

In calculations for a gas at STP, *conversions* using the STP prompt will often be the fastest way to solve. However, if gas conditions are at temperatures or pressures that are *not* STP, you will need to use *gas equations* to solve.

The gas equation encountered most frequently is the **ideal gas law** (often remembered as *piv-nerf*). For all “ideal” gases,

$$\boxed{PV = nRT}$$

where

- **P** and **V** are pressure and volume in any units;
- **T** is temperature in *kelvins*
- **n** represents the number of *moles* of gas; and
- **R** is a constant ratio: a number with ratio units called the **gas constant**.

In this equation, **P**, **V**, **T**, and **n** are **variables**. They can have any values, depending on the conditions in the problem.

The gas constant (**R**) is not a variable. It does not change as you vary **P**, **V**, **T** and **n**.

When using **R** to solve problems, **R** must have the same units as those used that are used to measure **P** and **V** in the problem. The *number* and *units* used for **R** will change depending on which **P** and **V** units are used. However, just as 12 fluid ounces is the same as 355 mL (our soda-can equality), the different numbers and units used for **R** do not change the constant *quantity* that **R** represents.

For any gas, if you know any three of the four variables in the ideal gas law, and you know a value and units for **R** (which you are usually supplied with the problem), you can use the ideal gas law to predict the fourth variable.

One of the interesting implications of the ideal gas law is that if any of the three variables in $PV=nRT$ are the same for samples of different gases, the fourth variable will have the same value even when the gases have very different molar masses and chemical formulas.

Non-Ideal Behavior

For most gases, the ideal gas law is a good *approximation* in predicting behavior, so long as the gas is not either at pressures substantially above one atmosphere *or* close to conditions of pressure and temperature where it condenses to form a liquid or solid. In reality, all real gases display “non-ideal” behavior if the temperature is low enough and/or the pressure is high enough. However, in the gas calculations in this module, you should assume that ideal gas conditions apply unless non-ideal behavior is specified.

Solving Problems Which Require Equations

Calculations in chemistry can generally be put into three categories: those that can be solved with conversions, those that require equations, and those requiring both.

We will use the gas laws to develop a *system* for solving calculations that require equations. This system will be especially helpful if you take additional science courses.

Let’s start with an easy problem, one in which we know the correct equation to use. This relatively easy example can be solved in several ways, but we will solve with a method that has the advantage of working with more difficult problems. Please, try the method used here.

For this problem, do the steps below in your notebook.

- Q.** A table of R values is usually supplied when solving gas problems. However, R can also be easily calculated for any set of units, based on values you know for standard temperature and pressure and the volume of a gas at STP.

For example, one mole of any gas occupies a volume of 22.4 liters at a standard pressure of *one atmosphere* and standard temperature (273 K).

Use this data and the ideal gas law equation to calculate a value for the gas constant (R), using the units specified in this problem.

Steps For Solving Equations

1. It says to use the ideal gas law. *Write* the equation from memory.
2. Next, make a data table that includes each *symbol* in the equation.

DATA: P =
 V =
 n =
 R =
 T =

3. Put a ? after the *symbol* WANTED in the problem.
4. Read the problem again, and write each number and unit after a symbol. Use the *units* to match the symbol to the data (one *mole* goes after **n**, 273 **K** goes after **T**, etc.).

Do those steps, then check below.

★ ★ ★ ★ ★

At this point, your paper should look like this.

$PV = nRT$

DATA: P = 1 atm
 V = 22.4 L
 n = 1 mole
 R = ?
 T = 273 K

5. Write SOLVE, and, using algebra, solve the fundamental, memorized equation for the symbol WANTED.

To simplify problem solving, the rule is:

Memorize equations in *one* fundamental format, then use algebra to solve for the symbol WANTED.

For example, memorize: $K = ^\circ C + 273$ Do *not* memorize $^\circ C = K - 273$

If Celsius is WANTED, knowing kelvins, write the memorized equation above, then solve the equation for Celsius.

Do not plug in numbers until after you have solved for the WANTED symbol in symbols.

Symbols move more quickly (and with fewer mistakes) than numbers with their units.

6. Plug in the numbers and solve. Cancel units when appropriate, but leave the units that do not cancel, and include them after the number that you calculate for the answer.

Do those steps, then check your work below.

* * * * *

On your paper, you should have added this:

SOLVE: PV = nRT

$$? = R = \frac{PV}{nT} = \frac{(1 \text{ atm})(22.4 \text{ L})}{(1 \text{ mol})(273 \text{ K})} = 0.0821 \frac{\text{atm} \bullet \text{L}}{\text{mol} \bullet \text{K}}$$

Be certain that your answer includes both numbers and units.

A check inside the cover of your class textbook may show that this answer is accurate to within one doubtful digit for one of the values listed for R.

Values for R vary depending on the units used to measure pressure and volume. If you need a value for R and you do not have access to a table, use the following rule as applied in the problem above.

To calculate a value for R using a required set of units, use PV=nRT, one mole of gas, and the values for P, V, and T at STP in the required units.

* * * * *

Summary: To solve an equation when you know which equation is needed.

1. Write the memorized equation.
2. Make a data table. On each line, put one symbol from the equation.
3. Based on units, after each symbol, write the matching DATA in the problem.
4. Solve your memorized equation for the WANTED *symbol* before plugging in numbers and units.
5. Put both numbers and units into the equation when you solve.

If units cancel to give an answer unit appropriate for the WANTED symbol, it is a check that the algebra was done correctly. If the units do not cancel properly, check your work.

Practice: Do them all.

1. Assign each of these gas measurements with a *symbol* from $PV = nRT$.
 - a. 0.50 moles
 - b. 202 kPa
 - c. 11.2 dm³
 - d. 373 K
 - e. 38 torr
2. Solve $PV = nRT$ for
 - a. $n =$
 - b. $T =$
 - c. $V =$
3. If the pressure of one mole of gas at STP increases to 2.3 atmospheres but the volume of the gas is held constant, what must the new temperature be?
 (To solve, use your calculated value above for R ($R = 0.0821 \text{ atm}\cdot\text{L}/\text{mol}\cdot\text{K}$), the ideal gas law, and the method used in this lesson.

ANSWERS

1. a. 0.50 moles n b. 202 kPa P c. 11.2 dm³ V d. 373 K T e. 38 torr P
2. If $PV = nRT$: (a) $n = \frac{PV}{RT}$ (b) $T = \frac{PV}{nR}$ (c) $V = \frac{nRT}{P}$

If you cannot do this algebra correctly *everytime*, find a friend or tutor who can help you to review the algebra for this and the following lessons in this module. It will not take long to master. Gas laws are not difficult if you can do this algebra but are impossible if you cannot.

3. $PV = nRT$

DATA: $P = 2.3 \text{ atm}$
 $V = 22.4 \text{ L}$
 $n = 1 \text{ mol}$

$$R = 0.0821 \frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}}$$

$$T = ?$$

$$\text{SOLVE: } PV = nRT$$

$$? = T = \frac{PV}{nR} = \frac{(2.3 \text{ atm})(22.4 \text{ L})}{(1 \text{ mole}) \left(0.0821 \frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}}\right)} = \frac{(2.3 \text{ atm})(22.4 \text{ L})}{1 \text{ mol}} \cdot \frac{\text{mol} \cdot \text{K}}{0.0821 \text{ atm} \cdot \text{L}} = 630 \text{ K}$$

630 K is a large rise in temperature from 273 K. It is reasonable because for one mole of gas at 22.4 liters volume to cause a pressure of 2.3 atm (rather than 1 atm, as at STP), the gas must be very hot, with molecules moving much faster on average than at standard temperature.

The unit cancellation: From Lesson 16C, use Rule 2a: Separate a fractional unit on the bottom into a reciprocal, and Rule 2c: A *fraction* on the bottom flips over when you bring it to the top.

Mark the unit cancellation in the final step above. When solving with equations, always include the unit cancellation. If the units cancel correctly, the numbers were probably put in the right place to get the right answer.

* * * * *

Lesson 17E: Choosing Consistent Units

Consistent Units

When solving *most* equations in science, numbers must have units attached, and the units must cancel to result in the unit WANTED. In order for units to cancel, the units must be **consistent**. This means

For each fundamental or derived quantity used in an equation, you must choose *one* unit to measure the quantity, then convert all data for that quantity to that unit.

For example, as one part of solving calculations using $PV=nRT$, you must pick a *pressure* unit (such as atmospheres, pascals, or torr), and convert all DATA that involves pressure to the unit you choose.

In some cases, an equation will require certain units.

For example, gas law equations using a capital T require temperature to be measured in an absolute temperature scale. In the metric system, this means *kelvins*. A DATA temperature that is not in kelvins must be converted to kelvins. If degrees Celsius is WANTED, kelvins must be found first.

When using $PV = nRT$, consistent units are a factor

- when you must pick an R value to use, and
- when you must convert DATA to match the units of R.

Let's take these cases one at a time.

Choosing an R

The gas constant R is one quantity, but it can be expressed in different units. Some of the equivalent values for R are in the table at the right.

$R = 0.0821 \text{ atm}\cdot\text{L}/\text{mol}\cdot\text{K}$ $= 8.31 \text{ kPa}\cdot\text{L}/\text{mol}\cdot\text{K}$ $= 62.4 \text{ torr}\cdot\text{L}/\text{mol}\cdot\text{K}$
--

Note that the *number* in each of those R values depends on the units used to measure P and V. However, just as a speedometer shows that 55 miles/hour is the same as 88 kilometers/hour, the different values for R do not change the *ratio* of the dimensions that R measures.

In gas calculations, you generally will not be required to memorize values for R, but you will need to select from a list or table which R to use. If you need to choose an R, the rule is:

Pick the R that has <i>units</i> that most closely match the units in the DATA.

When solving ideal gas law calculations, R, P, V, and T must have **consistent units**: the units used to measure P, V and T must match the units attached to the chosen R. In some problems, this will require converting the units of P, V, and/or T in the DATA table to the units of R, but if the R chosen from the table above has units that are a close match to the units in the supplied DATA, the conversions needed for P, V, and T will be minimized.

Keeping those factors in mind, try the following problem in your notebook.

Q. A sample of an ideal gas at 293 K and 202 kPa has a volume of 301 mL. How many moles are in the sample? (Use one of the R values above.)

* * * * *

The WANTED unit has the symbol **n**, 293 K is a **T**, 202 kPa is a **P**, and 301 mL is a **V**. What equation relates **n**, **T**, **P**, and **V**?

* * * * *

$PV = nRT$

Complete the DATA table using those symbols.

* * * * *

DATA: P = 202 kPa
 V = 301 mL
 n = ? mol = WANTED
 R = ?
 T = 293 K

Which R value should you choose?

* * * * *

R = 8.31 **kPa**•L/mol•K is not an exact match with the DATA units, but because it uses *kPa* it is the closest. Add that R value to the DATA table.

One more change is needed. The units for P, T, n, match the chosen R units, but our best match for R uses liters, while the supplied V is in mL. The volume unit must be consistent: we must pick mL or L, or the units will not cancel and the equation will not work. Since we don't see an R value that uses mL, what's the best option?

* * * * *

Convert the supplied DATA to liters in the DATA table, then complete the problem.

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DATA: P = 202 kPa
 V = 301 mL = **0.301 L** (by inspection, see Lesson 12A)
 n = ? moles = WANTED
R = 8.31 kPa•L/mol•K
 T = 293 K Are all of the units now consistent?

SOLVE: PV = nRT for the WANTED symbol *then* plug in numbers and units.

★ ★ ★ ★ ★

$$? = n = \frac{PV}{RT} = \frac{(202 \text{ kPa})(0.301 \text{ L})}{(293 \text{ K}) \cdot 8.31 \frac{\text{kPa} \cdot \text{L}}{\text{mol} \cdot \text{K}}} = \frac{(202 \text{ kPa})(0.301 \text{ L})}{(293 \text{ K})} \cdot \frac{\text{mol} \cdot \text{K}}{8.31 \text{ kPa} \cdot \text{L}} =$$

$$= \mathbf{0.0250 \text{ mol}}$$

Check that the units cancel properly. They must. If you have done the problem correctly, they will.

Practice A: Use one of the following values.

$$\mathbf{R = 0.0821 \text{ atm} \cdot \text{L} / \text{mol} \cdot \text{K} = 8.31 \text{ kPa} \cdot \text{L} / \text{mol} \cdot \text{K} = 62.4 \text{ torr} \cdot \text{L} / \text{mol} \cdot \text{K}}$$

1. If 0.0500 mole of an ideal gas at 0°C has a volume of 560. mL, what will be its pressure in torr?

Converting to Consistent Units

In many science calculations, we are given only one version of a constant. In such cases, the constant can be converted to other units, but it is usually easier to convert the DATA to the units of the constant.

In solving with equations, once you have chosen the *unit* that you will use for each equation symbol, a systematic approach is to write those units in the DATA table after each symbol, such as

DATA: P **in atm** =
 V **in L** =
 T **in K** =

Then list the data as supplied, and, in the DATA table, convert the supplied data to the consistent unit written after the variable symbol.

Try that approach in this problem.

- Q.** Find the temperature in degrees Celsius of 0.0100 moles of an ideal gas that has a volume of 125. mL and a pressure of 2.00 atm. (USE R = 8.31 kPa•L/mol•K)

Decide the equation that will be used, then make a DATA table using the symbols in the equation. After each symbol, write the chosen consistent unit, then an = sign, as done

above. In this problem, choose as consistent units the units in the supplied R. Since this R will decide the remaining units, put R and its value in the DATA table first.

* * * * *

The wanted unit has the symbol **T** and the data is in moles (**n**), mL (**V**) and pressure (**P**). The equation relates **T**, **n**, **V** and **P**?

$$PV = nRT$$

DATA: **R = 8.31 kPa•L/mol•K** (required in this problem)

$$P \text{ in kPa} = 2.00 \text{ atm}$$

In this problem, *P must* be in kPa to be consistent with the required R units. Convert the P data to kPa, then finish the table.

* * * * *

$$P \text{ in kPa} = 2.00 \text{ atm} \bullet \frac{101 \text{ kPa}}{1 \text{ atm}} = \mathbf{202 \text{ kPa}}$$

$$V \text{ in L} = 125 \text{ mL}$$

V must be in L to be consistent with the R units. Convert V, finish the table, and solve.

* * * * *

$$V \text{ in L} = 125 \text{ mL} = \mathbf{0.125 \text{ L}} \quad (\text{by inspection: mL is } 1000\times \text{ L})$$

$$n \text{ in mol} = 0.0100 \text{ mol}$$

T must always be in **K** but °C is WANTED

For temperature, we want Celsius, but the equation only works in kelvins. When using equations, the rule is: solve for the WANTED quantity in consistent or required units first, then convert if needed to other WANTED units.

SOLVE: $PV = nRT$ for the WANTED symbol *then* plug in numbers and units.

* * * * *

$$T = \frac{PV}{nR} = \frac{(202 \text{ kPa})(0.125 \text{ L})}{(0.0100 \text{ mol})(8.31 \frac{\text{kPa} \cdot \text{L}}{\text{mol} \cdot \text{K}})} = \frac{(202 \text{ kPa})(0.125 \text{ L})}{(0.0100 \text{ mol})} \bullet \frac{\text{mol} \cdot \text{K}}{8.31 \text{ kPa} \cdot \text{L}} = \mathbf{304 \text{ K}}$$

If the WANTED unit is what it should be after unit cancellation, there is a good chance that the numbers were put in the right place to get the right answer.

Finally, since the WANTED unit Celsius, use the equation that relates K and °C.

$$\boxed{K = ^\circ\text{C} + 273} \quad \text{so} \quad ^\circ\text{C} = K - 273 = 304 \text{ K} - 273 = \mathbf{31 \text{ } ^\circ\text{C}}$$

To summarize:

If an equation has a constant that includes units, and the constant must be used,

- In the DATA table, list the *constant* first.
- After each symbol for a *variable*, write a *unit* to convert DATA to. Choose if available a unit that matches the *constants* of the equation.
- First SOLVE in the *consistent* unit, then convert to the WANTED unit if needed.

Practice B:

1. If 2.00 moles of Ne gas has a pressure of 0.500 atm and a temperature of 25 °C, what will be its volume in liters? (USE: $R = 8.31 \text{ kPa}\cdot\text{L}/\text{mol}\cdot\text{K}$)

ANSWERS

Practice A

1. $PV = nRT$

DATA: **P in torr = WANTED**

$$R = 62.4 \frac{\text{torr}\cdot\text{L}}{\text{mol}\cdot\text{K}} \quad (\text{the only R value that uses torr})$$

$$V \text{ in L to match R unit} = 560. \text{ mL} = \mathbf{0.560 \text{ L}} \quad (\text{mL to L: see Lesson 12A})$$

$$n = \mathbf{0.0500 \text{ mol}}$$

$$T \text{ must be in K: } \boxed{K = ^\circ\text{C} + 273} = 0^\circ\text{C} + 273 = \mathbf{273 \text{ K}}$$

SOLVE: $PV = nRT$

$$? = P = \frac{nRT}{V} = nRT \cdot \frac{1}{V} = (0.0500 \text{ mol})(62.4 \frac{\text{torr}\cdot\text{L}}{\text{mol}\cdot\text{K}})(273 \text{ K}) \cdot \frac{1}{0.560 \text{ L}} = \mathbf{1,520 \text{ torr}}$$

Mark the unit cancellation in the last step.

To help with unit cancellation, a good rule is

When solving an equation in *symbols* results in a *fraction* that include terms with *fractional* units (such as **R** above) on the top *or* bottom, re-write the equation with the *symbols* in the denominator separated into a reciprocal, then plug in numbers into this separated format (see Lesson 17C).

However, you may do the math for the numbers and units in any way you choose, provided you do *both* numbers *and* units.

Practice B

1. $PV = nRT$

DATA: **V in liters = WANTED**

$$R = 8.31 \frac{\text{kPa}\cdot\text{L}}{\text{mol}\cdot\text{K}} \quad (\text{If a certain constant must be used, write it first})$$

$$P \text{ in kPa} = 0.500 \text{ atm} \bullet \frac{101 \text{ kPa}}{1 \text{ atm}} = 50.5 \text{ kPa}$$

(↑ convert P in data to kPa to be consistent with the units of R)

V in liters = WANTED

$$T \text{ must be in K : } \boxed{K = ^\circ\text{C} + 273} = 25 ^\circ\text{C} + 273 = 298 \text{ K}$$

SOLVE: $PV = nRT$

$$? = V = \frac{nRT}{P} = nRT \bullet \frac{1}{P} = (2.00 \text{ mol}) \left(\frac{8.31 \text{ kPa} \cdot \text{L}}{\text{mol} \cdot \text{K}} \right) (298 \text{ K}) \bullet \frac{1}{50.5 \text{ kPa}} = 98.1 \text{ L}$$

* * * * *

Lesson 17F: Density, Molar Mass, and Choosing Equations

The ideal gas law has one constant (R) and 4 variables (P, V, T, and n). The $PV = nRT$ equation may be *linked* to other equations to find other variables. The key to this *relating* of equations is to find a variable which the equations share.

In chemistry, the variable that most relationships share most often is the unit for counting the *number of particles* involved in a chemical process: *moles*.

Grams, Molar Mass, and Ideal Gases

Knowing any four of the quantities in the equation $PV = nRT$, the fifth can be calculated.

For the grams, *moles*, and molar mass of any substance, if you know any two of those quantities, conversions will find the third.

Because *moles* is a common factor in both of those two relationships, $PV = nRT$ can be linked to calculations involving grams and molar mass.

In calculations involving *grams* and *molar mass* with the ideal gas law, a useful strategy is to solve in two parts, solving for the common variable (moles) in one part, then using that answer to solve the other part.

- Find moles using whichever relationship, either $PV = nRT$ or the grams to moles conversion, provides enough data to find moles. If the final WANTED variable is in *one* part, you will usually need to solve the *other* part first.
- Then use the found moles as DATA in the other part to solve for the variable WANTED.

Keeping those two steps in mind, try this example in your notebook.

Q. A sample of gas has a volume of 5.60 liters at 2.00 atm pressure and standard temperature. If the gas sample has a mass of 2.00 grams, what is the molar mass of the gas? ($R = 0.0821 \text{ atm} \cdot \text{L} / \text{mol} \cdot \text{K} = 8.31 \text{ kPa} \cdot \text{L} / \text{mol} \cdot \text{K} = 62.4 \text{ torr} \cdot \text{L} / \text{mol} \cdot \text{K}$)

* * * * *

Answer

It says to use R. So far, we only know one relationship involving R.

$$PV = nRT$$

Knowing the specific equation needed, use those symbols for the DATA table.

DATA: $P = 2.00 \text{ atm}$

$V = 5.60 \text{ L}$

$n = ?$

$R = 0.0821 \frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}}$ (consistent units: P uses atm)

$T = 273 \text{ K}$ (std. T)

This problem has *additional* WANTED and DATA:

WANT: Molar Mass = $? \frac{\text{g}}{\text{mol}}$

DATA: 2.00 g gas

The final WANTED unit is a grams to moles ratio. The data supplies grams. IF the moles of the gas can be found, dividing grams by moles finds the molar mass.

In the $PV=nRT$ part of the problem, 4 of the 5 quantities are known; algebra will find the 5th, the *moles* needed to find the molar mass.

The strategy? Solve for the linked variable first in the part of the calculation that does not include the final WANTED unit, then use that answer to solve the relationship that includes the final WANTED unit.

SOLVE: $PV = nRT$ You want to find moles.

$$? = n = \frac{PV}{RT} = \frac{(2.00 \text{ atm})(5.60 \text{ L})}{(0.0821 \frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}})(273 \text{ K})} = \boxed{0.5000 \text{ moles}}$$

(In calculations that use an answer from one part to solve a later part, carry an extra *sf* until the final step.)

Use the answer for that part as DATA to find the WANTED.

$$\text{WANTED: } ? \frac{\text{g}}{\text{mol}} = \frac{2.00 \text{ g}}{\boxed{0.5000 \text{ mol}}} = 4.00 \frac{\text{g}}{\text{mol}}$$

Practice A: Use the above methods. Answers are at the end of this lesson.

($R = 0.0821 \text{ atm} \cdot \text{L} / \text{mol} \cdot \text{K} = 8.31 \text{ kPa} \cdot \text{L} / \text{mol} \cdot \text{K} = 62.4 \text{ torr} \cdot \text{L} / \text{mol} \cdot \text{K}$)

1. If a 1.76 gram sample of uranium hexafluoride gas is at 380. torr and 600. K, what volume will it occupy?
2. A sample of neon gas has a volume of 8.96 L at 298 K and 2.00 atm. How many grams of Ne are in the sample?

Density, Molar Mass, and Ideal Gases

The relationship between the ideal gas law, molar mass, and gas *density* is more mathematically complex than the relationship involving grams and moles. For a complex relationship, it is often best to derive an equation, then memorize it.

The relationship using density and molar mass for ideal gases can be derived as follows.

$$\text{Molar Mass (MM)} \equiv \frac{\text{grams}}{\text{moles}}, \text{ so we can write } \boxed{\text{grams} = \text{moles} \cdot \text{MM}} \quad (1)$$

$$\text{Density (D)} \equiv \frac{\text{mass}}{\text{volume}}; \text{ using grams for mass, } \boxed{D_{\text{grams}} = \frac{\text{grams}}{\text{Any V unit}}} \quad (2)$$

Substituting equation (1) into (2),

$$D_{\text{grams}} = \frac{\text{moles} \cdot \text{MM}}{V}, \text{ which can be rewritten as } \frac{\text{moles}}{V} = \frac{D_{\text{grams}}}{\text{MM}} \quad (3)$$

Since $PV = nRT$ can be re-written as $P = \frac{\text{moles}}{V} \cdot RT$,

$$\text{Substituting with equation (3) gives } \boxed{P = \frac{D_{\text{grams}} \cdot RT}{\text{MM}}} \quad (4)$$

The density can be measured using any mass unit as long as the in mass/mol ratio of the molar mass, the mass is measured in the same unit.

Equation (4) is a form of the ideal gas law that uses density and molar mass in place of moles and volume. You can either work out the above derivation every time an ideal gas problem involves density..., OR you can *memorize* these *two* equations:

$$\boxed{\text{The ideal gas law: } PV = nRT \quad \text{and} \quad P = \frac{D_{\text{grams}} \cdot RT}{\text{MM}}}$$

If an ideal gas problem mentions gas density or molar mass at conditions *not* at STP, it is usually a prompt to use the form of the ideal gas law that includes density and molar mass.

When the Equation To Use Is Not Clear

So far, we have learned only two gas-law equations, but we are about to learn a few more. Other science courses will have *many* equations to memorize. How do you decide *which* equation to use *when*?

Correctly choosing which to use will require a *system*. The following system is easy to use, and it will work in both chemistry and other science courses.

A System for Finding the Right Equation

When you are not sure *which* equation to use or *whether* to use equations or conversions to solve, do these steps.

1. As you have been doing, first write the WANTED *unit* and/or *symbol*.
2. List the DATA with units, substances, and descriptive labels. Add any prompts.

- Analyze whether the problem will require *conversions* or an *equation*.
 - Try conversions first. Conversions often work if *most* of the data are in *pairs* and *ratio units*.
 - If the data is mostly in single units, you will likely need an equation. Recent lessons in your course will likely indicate the equations that may be needed.
 - Watch for hints at the need for an equation. For example, the mention of **R** in a gas problem is likely a prompt that you will need a form of the ideal gas law to solve.
- If a lecture or textbook is frequently using certain equations, learn them *before* you do the practice problems. Write them at the top of your paper at the start of each assignment, quiz, or test.
- If you recognize which equation is needed, write the equation, then make a DATA table listing every symbol in the equation. Fill in the table with the DATA from the problem.
- If conversions don't work and you *suspect* you need an equation, but you are not sure *which* equation is needed, try this:

Label each item of WANTED and DATA with a *symbol* based on its units. Use symbols that are used in the equations for the topic you are studying.

For example,

- 25 **kPa** would be labeled with a **P** for pressure;
- 293 **K** would get a **T** for Kelvin scale temperature;
- 20^o**C** gets a lower case **t** for degrees Celsius;
- Liters or mL or cm³ or dm³ would be assigned a **V** for volume;
- 25 **grams** would be labeled with a lower case **m** for mass;
- 18.0 **grams per mole** -- label **MM** for molar mass;
- 2.5 **moles per liter**, in a problem about aqueous solutions, would be labeled **M** for molarity;
- 7.5 grams per liter, a **mass over volume**, would be labeled **D** for density.

Label the WANTED unit, as well as each item of DATA, with a symbol.

- Compare* the symbols listed in the WANTED and DATA to your written, memorized list of equations for the topic. Find the equation that *uses* those symbols. Write the equation, as memorized, below the data.

If no equations match exactly, see if the problem's symbols can be converted to give the symbols for a known equation (for example, degrees Celsius can be converted to kelvins, and grams can be converted to moles if you know a substance formula).

- Watch for the variables that make *similar* equations different. For example, in the two forms of the ideal gas law, the first uses volume and moles. The second uses molar mass and density instead of volume and moles. The *symbols* that you assign to the WANTED and DATA will identify which equation to use.

9. SOLVE the equation for the WANTED variable in symbols *before* plugging in numbers. Symbols move quickly. If instead of moving symbols, you move numbers *and* their units *and* their labels, more errors will tend to occur.

Practice B

Using the method above, try these in your notebook. If you get stuck, read a portion of the answer and try again.

- Assign symbols used in gas law equations to these quantities.
 - 122 g/mol
 - 202 kPa
 - 13.5 g/mL
 - 30°C
- Solve $P = \frac{D_{\text{grams}} \cdot RT}{MM}$ for a) D = b) MM c) T
- If the density of a gas at 27°C and standard pressure is 1.79 g/L, what is its molar mass?
- Write values for standard pressure in 5 different units.
- If one mole of any gas occupies 22.4 liters of volume at STP, calculate the value of R in units of kPa • L/mol • K. (Do not use a table value for R.)
- What is the density of hydrogen gas (H₂) in grams per liter at STP?

ANSWERS

Practice A

- Hints:
 - The data will include 1.76 g UF₆
 - It says to use R. Write the only relationship that we know so far which uses R.

* * * * *

$$PV = nRT$$

Make your data table to match the equation:

DATA:	P = 380 torr	n = ?	
	V = ? WANTED	R = 62.4 $\frac{\text{torr} \cdot \text{L}}{\text{mol} \cdot \text{K}}$	(P uses torr)
		T = 600. K	

Plus: 1.76 g UF₆
 352.0 g UF₆ = 1 mol UF₆ (g prompt)

Strategy: Looking at the PV=nRT data table, if all of the quantities but one are known, we can solve with algebra. Missing 2 variables, we can't. But the *plus* data gives us a way to find *moles* of the gas. The rule is: first solve the part without the final WANTED variable.

$$? \text{ mol UF}_6 = 1.76 \text{ g UF}_6 \bullet \frac{1 \text{ mol UF}_6}{352.0 \text{ g UF}_6} = 5.000 \times 10^{-3} \text{ mol UF}_6$$

Adding those moles to the PV=nRT data table, the V WANTED can be found.

$$? = V = \frac{nRT}{P} = nRT \cdot \frac{1}{P} = \left(5.000 \times 10^{-3} \text{ mol}\right) \left(\frac{62.4 \text{ torr} \cdot \text{L}}{\text{mol} \cdot \text{K}}\right) (600 \text{ K}) \cdot \frac{1}{380 \text{ torr}} = \mathbf{0.493 \text{ L}}$$

2. WANT: ? g Ne

DATA: $PV = nRT$ (It says to use a relationship with R)

$$P = 2.0 \text{ atm} \quad V = 8.96 \text{ L} \quad n = ?$$

$$R = 0.0821 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \quad \text{(Pick an R value that has units that match the units used in the problem.)}$$

$$T = 298 \text{ K}$$

Plus: 20.2 g Ne = 1 mol Ne (grams prompt)

Strategy: You want grams. If you know the moles, you can find the grams by molar mass conversion. Using $PV=nRT$, you can find the moles. Complete that part first.

$$? = n = \frac{PV}{RT} = \frac{PV}{T} \cdot \frac{1}{R} = \frac{2.00 \text{ atm} \cdot 8.96 \text{ L}}{298 \text{ K}} \cdot \frac{\text{mol} \cdot \text{K}}{0.0821 \text{ L} \cdot \text{atm}} = \mathbf{0.7324 \text{ mol Ne}}$$

Note how the term with the fractional unit (R) was separated to simplify cancellation.

Use the common variable from one part to solve the other part.

* * * * *

$$? \text{ g Ne} = 0.7324 \text{ mol Ne} \cdot \frac{20.2 \text{ g Ne}}{1 \text{ mol Ne}} = \mathbf{14.8 \text{ g Ne}}$$

Practice B

1. a. 122 g/mol **MM** b. 202 kPa **P** c. 13.5 g/mL **D** d. 30°C **t**

2. Solve $P = \frac{D_{\text{grams}} \cdot RT}{MM}$ for a) $D = \frac{P \cdot MM}{RT}$

b) $MM = \frac{D_{\text{grams}} \cdot R \cdot T}{P}$ c) $T = \frac{P \cdot MM}{D_g \cdot R}$

3. Use conversions or equations? To decide, list the data. If the data is mostly pairs or ratio units, try conversions. If conversions won't work (and on this problem they don't), use the units and words in the problem to assign symbols, and see if a known formula fits the symbols.

WANTED: ? $\frac{\text{g}}{\text{mole}}$ **MM**

DATA: 27°C **t** **T** = 27°C + 273 = 300. K (3 sf – doubt in one's place when adding)

Std. P = 1 atm **P**

1.90 g = 1 L **Dgrams**

(Strategy: If the gas were at STP, we could use conversions to solve, but 27°C is not standard temperature.

We have memorized two gas equations that use pressure and temperature:

$$PV = nRT \quad \text{and} \quad P = \frac{D_{\text{grams}} \cdot RT}{MM}$$

The second equation fits the data. (R is a constant that you are normally allowed to look up in a table even if it is not supplied (and if not, it is easy to calculate)).

If an equation fits the symbols, try the equation. Solving for the wanted MM:)

$$MM = \frac{\text{density}}{P} \cdot RT = 1.79 \frac{\text{g}}{\text{L}} \cdot \frac{1}{1 \text{ atm}} \cdot 0.0821 \frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}} \cdot 300. \text{ K} = 44.1 \frac{\text{g}}{\text{mol}}$$

Terms with fractional units (density and R) were separated for cancellation.

4. 1 Atmosphere \equiv 760 mm Hg \equiv 760 Torr = 101 kilopascals (kPa) = 1.01 bars
5. WANTED: **R** in $\frac{\text{kPa} \cdot \text{L}}{\text{mol} \cdot \text{K}}$

(R is in both “ideal gas law” equations. To decide which to use, list and *symbol* the data.)

DATA: std P = **101 kPa** in the units wanted for R **P**

22.4 L **V**

1 mol **n**

std. temp = 273 K **T**

Strategy: Looking at **all 5** symbols labeling WANTED *and* DATA, which equation works?

SOLVE: **PV = nRT**

$$? = R = \frac{PV}{nT} = \frac{(101 \text{ kPa})(22.4 \text{ L})}{(1 \text{ mol})(273 \text{ K})} = 8.29 \frac{\text{kPa} \cdot \text{L}}{\text{mol} \cdot \text{K}}$$

Using less rounded values for P, T, and V, the accepted value is 8.31 kPa \cdot L / mol \cdot K

6. WANTED: ? $\frac{\text{g H}_2 \text{ gas}}{\text{L H}_2 \text{ gas at STP}}$

DATA: 2.016 g H₂ = 1 mol H₂ (grams prompt)

1 mol any gas = 22.4 L any gas at STP (STP Prompt)

Strategy: Analyze your units. Equalities lend themselves to conversions. You want grams over liters. You know grams to moles and moles to liters.

SOLVE:

$$? \frac{\text{g H}_2 \text{ gas}}{\text{L H}_2 \text{ gas at STP}} = \frac{2.016 \text{ g H}_2}{1 \text{ mol H}_2} \bullet \frac{1 \text{ mol gas}}{22.4 \text{ L gas STP}} = 0.0900 \frac{\text{g H}_2(\text{g})}{\text{L H}_2 \text{ at STP}}$$

* * * * *

Lesson 17G: The Combined Equation

Many gas-law calculations involve the special case where the gas is trapped (the *moles* of gas in a sample does *not* change) while *P*, *V* and/or *T* are changed.

If the number of gas moles is held constant, we can rewrite $PV = nRT$ as

$$\frac{PV}{T} = nR = (\text{constant moles})(\text{gas constant } R) = (\text{a new constant})$$

When you multiply two constants, the result is new constant. While *R* is always a constant, the new constant above will only be true for the number of moles in a problem.

The equation above means that if gas conditions are changed while the moles of gas are held constant, the ratio “*P* times *V* over *T*” will keep the same numeric value no matter how you change *P*, *V*, and *T*.

Another way to express this relationship: so long as the number of particles of gas does not change, if you have an *initial* set of conditions P_1 , V_1 , and T_1 , and you change to a *new* set of conditions P_2 , V_2 , and T_2 , the ratio PV/T must stay the same. Expressed in the elegant and efficient shorthand that is algebra,

$$nR = \boxed{\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}} \quad \text{when the moles of gas is held constant.}$$

The boxed equation means that IF 5 of the 6 variables among P_1 , V_1 , T_1 , P_2 , V_2 , and T_2 are known, the 6th variable may be found using algebra, *without* knowing **n** or needing **R**.

We will call this relationship the **combined equation** (because it combines three historic gas laws). You may also see it referred to as the **two-point equation** since it is based on *initial* and *final* conditions. For problems in which the moles of gas particles do not change, when one set of conditions is changed to new conditions, the quickest way to solve is usually to apply

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \text{where } P_1, V_1, \text{ and } T_1 \text{ are } \textit{starting} \text{ gas measurements,}$$

$$\quad \quad \quad \text{and } P_2, V_2, \text{ and } T_2 \text{ are the } \textit{final} \text{ gas measurements.}$$

and the moles of gas stay the same.

This equation is often memorized by repeated recitation of “*P* one *V* one over *T* one equals *P* two *V* two over *T* two.”

It may help to remember this rule:

When a problem says a sample of gas is *sealed* or *trapped* or has *constant moles*, and the sample has a change in conditions, see if the *combined equation* symbols fit the data.

To solve a calculation in which the equation needed is known, apply the system developed in the previous lesson to solve equation calculations.

- **Write** the fundamental equation.
- Set up a **data** table that contains the **symbols** in the equation.

In a problem that requires the combined equation, start with

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$	(when the moles of gas are held constant)
DATA: $P_1 =$	$P_2 =$
$V_1 =$	$V_2 =$
$T_1 =$	$T_2 =$

Then,

- put the numbers and their units from the problem into the table.
 - Label the symbol WANTED with a ?. Add the units WANTED if they are specified.
 - SOLVE the fundamental equation in *symbols* for the WANTED symbol, using algebra, *before* you plug in numbers.
 - *Then* plug in numbers *and* units, and solve. Apply unit cancellation as well as number math. Include the units that do *not* cancel with your answer.
 - Try the method on this problem, using the combined equation to solve:
- Q.** If a sample of gas in a sealed but flexible balloon at 273K and 1.00 atm pressure has a volume of 15.0 liters, and the pressure is increased to 2.5 atmospheres while the temperature is increased to 373 K, what will be the new volume of the balloon?

* * * * *

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

DATA: $P_1 = 1.0 \text{ atm}$	$P_2 = 2.5 \text{ atm}$
$V_1 = 15.0 \text{ Liters}$	$V_2 = ?$
$T_1 = 273 \text{ K (std T)}$	$T_2 = 373 \text{ K}$

SOLVE: $? = V_2 = \frac{P_1 V_1 T_2}{P_2 T_1} = \frac{(1.0 \text{ atm})(15.0 \text{ L})(373 \text{ K})}{(2.5 \text{ atm})(273 \text{ K})} = 8.2 \text{ L}$

If the units do not cancel to give the WANTED unit, check your work.

Practice A: Check answers at the end of this lesson as you go.

1. Solve the combined equation for
 - a. $T_2 =$
 - b. $P_1 =$
 - c. $V_2 =$
2. A gas cylinder with a volume of 2.50 liters is at room temperature (293 K). The pressure inside the tank is 100. atmospheres. When the gas is released into a 50.0 liter container, the gas pressure falls to 2.00 atmospheres. What will be the new temperature of the gas in kelvins, and in degrees Celsius?

Consistent Units

In gas law equations, a *capital T* means *absolute* temperature. Because the combined equation is derived from $PV=nRT$, when using metric units, both of those equations *require* that

- temperatures must be converted to kelvins before substituting in the equation.
- If a temperature that is not kelvins is WANTED, kelvins must be solved first, and then converted to other temperature units.

For **P** and **V**, each variable must be converted to a *consistent* unit to solve. In most *combined* law calculations, since there is no R value with complex units that we are required to match, it does not matter which **V** or **P** units you choose.

- Volume may be in mL or L; and
- Pressure can be converted to kPa, atmospheres, torr, bars, or other pressure units.

However, *you* must choose *one unit* for P and for V, and all DATA for that quantity must be converted to that unit. It usually simplifies calculations if you convert DATA to the WANTED unit if it is specified, but any units will work in the equation as long as units are consistent. If non-WANTED units are found using the equation, simply convert those units to the units WANTED.

Keeping that in mind, try this problem.

- Q.** An aerosol-spray can contains 250. mL of gas under 4.5 atm pressure at 27°C. How many liters would the gas occupy at 50.5 kPa and standard (std.) T?

* * * * *

Answer: When one set of gas conditions is changed to new conditions, but moles are held constant, try

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

(If you need an equation, write it first. The more often you write it, the longer it is remembered.)

DATA: $P_1 = 4.5 \text{ atm}$	$P_2 = 50.5 \text{ kPa} = \mathbf{0.500 \text{ atm}}$
$V_1 = 250. \text{ mL} = 0.250 \text{ L}$	$V_2 = ? \text{ L WANTED}$
$T_1 \text{ in K} = 27 \text{ }^\circ\text{C} + 273 = 300. \text{ K}$	$T_2 \text{ in K} = 273 \text{ K} = \text{std. T}$

Unit Conversions:

For T_1 , *must* use K. $\boxed{K = ^\circ\text{C} + 273} = 27 \text{ }^\circ\text{C} + 273 = \mathbf{300. \text{ K}}$

For P_1 , choose one of the units in the DATA: *kPa* or *atm*. Either choice will result in the same answer. *If* you choose atm,

$$P_2 = ? \text{ atm} = 50.5 \text{ kPa} \bullet \frac{1 \text{ atm}}{101 \text{ kPa}} = \mathbf{0.500 \text{ atm}} \quad (\text{or } P_1 = 455 \text{ kPa})$$

Since liters is WANTED, and the initial volume is in mL, pick a unit and convert the other to it. It's best to pick the *answer* unit if it is specified.

$$? \text{ L} = V_1 = 250. \text{ mL} = \mathbf{0.250 \text{ L}} \quad (\text{by inspection: See Lesson 12A})$$

Solve for the WANTED symbol, *then* substitute the DATA.

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$$\text{SOLVE: } ? = V_2 = \frac{P_1 V_1 T_2}{P_2 T_1} = \frac{(4.5 \text{ atm}) (0.250 \text{ L}) (273 \text{ K})}{(0.500 \text{ atm}) (300. \text{ K})} = 2.0 \text{ L}$$

Practice B

1. A sealed sample of hydrogen gas occupies 500. mL at 20. °C and 150. kPa. What would be the temperature in degrees Celsius if the volume of the container is increased to 2.00 liters and the pressure is decreased to 0.550 atm?

Simplifying Conditions

In a calculation involving a change from initial to final conditions for a constant moles of gas (gas in a *sealed* container), the combined equation is used to solve. However, if temperature does not change, converting to kelvins is not necessary. Why?

If $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$, and $T_1 = T_2$, then $\frac{P_1 V_1}{\cancel{T_1}} = \frac{P_2 V_2}{\cancel{T_2}}$; use $P_1 V_1 = P_2 V_2$

If the “before and after” temperatures are the same, T is not needed to solve.

When using the *combined* equation, if *any* two symbols have the same value in a problem, those symbols can be cancelled because they are the same on both sides. Try this example in your notebook.

- Q.** A sample of chlorine gas has a volume of 22.4 liters at 27°C and standard pressure. What will be the pressure in torr if the temperature does not change but the volume is compressed to 16.8 L?

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Answer

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$	(Write needed fundamentals <i>before</i> you get immersed in details)
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DATA: $P_1 = \text{Std. P} - \text{use } 760 \text{ torr to match} \rightarrow$	$P_2 = ? \text{ in torr}$
$V_1 = 22.4 \text{ L}$	$V_2 = 16.8 \text{ L}$
$T_1 = 27^\circ\text{C} = T_2$	T_2

Solve for the WANTED symbol first, *then* substitute the DATA.

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$$\text{SOLVE: } \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} ; \quad P_1 V_1 = P_2 V_2$$

$$? = P_2 \text{ in torr} = \frac{P_1 V_1}{V_2} = \frac{(760 \text{ torr})(22.4 \text{ L})}{(16.8 \text{ L})} = 1,010 \text{ torr}$$

Practice C

1. A 0.500 liter sample of neon gas in a sealed metal container is at 30. °C and 380. mm Hg. What would be the pressure of the gas in kPa at standard temperature?

ANSWERS

Practice A

1. a. T_2 If $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$, then $T_2 = \frac{P_2 V_2 T_1}{P_1 V_1}$
 b. $P_1 = \frac{P_2 V_2 T_1}{T_2 V_1}$ c. $V_2 = \frac{P_1 V_1 T_2}{T_1 P_2}$

As a check, note the *patterns* above.

- The P's and V's, if in the same group, have the same subscript, but a T grouped with them will have the opposite subscript.
- In the fractions, there is always one more term on the top than on the bottom.
- If you substitute consistent units for all of the symbols, the units cancel correctly.

If you have trouble solving for any of the six symbols, find a friend or tutor to help you learn the algebra.

2. If the moles of gas particles do not change, and one set of conditions is changed to new conditions, use

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

DATA: $P_1 = 100. \text{ atm}$

$P_2 = 2.00 \text{ atm}$

$V_1 = 2.50 \text{ L}$

$V_2 = 50.0 \text{ L}$

$T_1 = 293 \text{ K}$

$T_2 = ? \quad t_2 = ?$

SOLVE: (In gas problems, to find Celsius, solve for kelvins first.)

$$? = T_2 = \frac{P_2 V_2 T_1}{P_1 V_1} = \frac{(2.00 \text{ atm})(50.0 \text{ L})(293 \text{ K})}{(100. \text{ atm})(2.50 \text{ L})} = 117 \text{ K}$$

To find Celsius:

$K = ^\circ\text{C} + 273$

$^\circ\text{C} = K - 273 = 117\text{K} - 273 = -156 \text{ }^\circ\text{C}$

Practice B

1. For trapped gas moles changing from original to new conditions, use

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

DATA:

$P_1 = 150. \text{ kPa}$

$P_2 = 0.550 \text{ atm} = 55.6 \text{ kPa}$

$V_1 = 500. \text{ mL}$

$V_2 = 2.00 \text{ L} = 2.00 \times 10^3 \text{ mL}$

$T_1 = 20 \text{ }^\circ\text{C} + 273 = 293 \text{ K}$

$T_2 = ? \quad t_2 = ?$

Needed Unit Conversions:

For T_1 , K must be used. $\boxed{K = ^\circ\text{C} + 273} = 20. \text{ }^\circ\text{C} + 273 = 293 \text{ K}$

For P_1 , kPa *or* atm could be used as units. If you choose kPa,

$$? \text{ kPa} = 0.550 \text{ atm} \bullet \frac{101 \text{ kPa}}{1 \text{ atm}} = 55.6 \text{ kPa}$$

SOLVE: (In gas problems, solve in kelvins first, then Celsius).

$$? = T_2 = \frac{P_2 V_2 T_1}{P_1 V_1} = \frac{(55.6 \text{ kPa})(2,000 \text{ mL})(293 \text{ K})}{(150. \text{ kPa})(500. \text{ mL})} = \boxed{434 \text{ K}}$$

To find Celsius, *first* write the memorized equation, *then* solve:

$$\boxed{K = ^\circ\text{C} + 273} \quad ^\circ\text{C} = K - 273 = 434 \text{ K} - 273 = \boxed{161 \text{ }^\circ\text{C}}$$

Practice C

1. For trapped gas changing to new conditions, use

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

DATA:

$P_1 = 380. \text{ mm Hg} = 50.5 \text{ kPa}$

$P_2 = ? \text{ kPa}$

$V_1 = 0.500 \text{ L} = V_2$

(a sealed metal container will have a constant volume)

$T_1 = 30.^\circ\text{C} + 273 = 303 \text{ K}$

$T_2 = 273 \text{ K}$

Needed Unit Conversions:For the pressures, you must change to consistent units: either kPa *or* mm Hg. If you choose kPa,

$$? \text{ kPa} = 380. \text{ mm Hg} \bullet \frac{101 \text{ kPa}}{760 \text{ mm Hg}} = 50.5 \text{ kPa}$$

SOLVE:
$$P_2 = \frac{P_1 \cancel{V_1} T_2}{\cancel{T_1} V_2} = \frac{(50.5 \text{ kPa})(273 \text{ K})}{(303 \text{ K})} = \boxed{45.5 \text{ kPa}}$$

* * * * *

Lesson 17H: Gas Law Summary and Practice

Gas Law Summary

If you have not already done so, master flashcards that include the following rules.

1. When measuring gases:

$$\begin{aligned} \text{Standard Pressure} &\equiv 1 \text{ atmosphere} \equiv 760 \text{ mm Hg (mercury)} \equiv 760 \text{ torr} \\ &= 101 \text{ kilopascals (kPa)} = 1.01 \text{ bars} \end{aligned}$$

$$\text{Standard Temperature} \equiv \text{zero degrees Celsius (} 0^{\circ}\text{C)} = 273 \text{ kelvins} = 273 \text{ K}$$

2. Temperatures must be converted to kelvins when using any equation which specifies a capital T.

$$\boxed{K = ^{\circ}\text{C} + 273}$$

3. To be a measure of moles, gas **volumes** must be labeled with a **P** and a **T**.

4. **The STP prompt.**

$\boxed{\text{If a gas is at STP, write as DATA: } 1 \text{ mole gas} = 22.4 \text{ liters gas at STP}}$

For gas calculations at STP, try the STP prompt and conversions first.

5. **The ideal gas laws:** $\boxed{PV = nRT}$ and

$$\boxed{P = \frac{D_{\text{grams}} \cdot RT}{MM}}$$

where **R** = the gas constant

$$= 0.0821 \text{ atm} \cdot \text{L/mol} \cdot \text{K} = 8.31 \text{ kPa} \cdot \text{L/mol} \cdot \text{K} = 62.4 \text{ torr} \cdot \text{L/mol} \cdot \text{K}$$

6. **The combined equation.** If the gas conditions are changed but the *moles* of gas does not change, use \rightarrow

$$\boxed{\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}}$$

7. To solve with equations, first convert to consistent units, solve in consistent units, then convert the consistent WANTED unit to other units if needed.

8. For solving *all kinds* of calculations, use a *system*.

- List the WANTED and DATA;
- Try solving with conversions first. If conversions do not work,
- Add symbols to the WANTED and DATA, and then write a memorized fundamental *equation* that *uses* those symbols.
- Convert all DATA to *consistent* units.
- Solve for the WANTED *symbol* before plugging in numbers.
- Cancel units as a check of your work.

9. For numbers, units, or symbols: $1/(1/X) = X$; $A/(B/C) = (A \cdot C)/B$

* * * * *

Practice: The problems below involve a variety of gas relationships. Using the rules above, you will have a *system* to solve. Try the odd problems, and some evens in a later study session. Problem 6 is more challenging.

$$(R = 0.0821 \text{ atm}\cdot\text{L}/\text{mol}\cdot\text{K} = 8.31 \text{ kPa}\cdot\text{L}/\text{mol}\cdot\text{K} = 62.4 \text{ torr}\cdot\text{L}/\text{mol}\cdot\text{K})$$

If you get stuck, read a part of the answer to get a hint, then try again.

- The gas in a sealed flexible balloon has a volume of 6.20 liters at 30.°C and standard pressure. What will be its volume at -10. °C and 740. torr?
- How many gas molecules will there be, per milliliter, for all gases at STP?
- If gas in a sealed glass bottle has a pressure of 112 kPa at 25°C, and the temperature of the gas is increased to 100.°C, what will be the pressure?
- If the density of a gas is 0.0147 g/mL at 20.°C and 1.00 atm pressure, what is its molar mass?
- In 70.4 grams of UF₆ gas, at a volume of 4.48 L and a pressure of 202.6 kPa, what is the temperature in degrees Celsius?
- Hydrofluoric acid can be used to etch glass. If 994 mL of HF gas at SP and 30.°C is dissolved in water to make 250. mL of HF acid solution, what is the [HF]?

ANSWERS

1. WANTED: ? V at end **V₂**

DATA: 6.20 L initial **V₁**

30.°C + 273 = 303 K initial **T₁**

std P = 760 torr initial, using the P units in the problem **P₁**

-10.°C + 273 = 263 K final **T₂**

740 torr final **P₂**

Strategy: The WANTED and DATA symbols match

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\text{SOLVE: } ? = V_2 = \frac{P_1 V_1 T_2}{P_2 T_1} = \frac{(760 \text{ torr})(6.20 \text{ L})(263 \text{ K})}{(740. \text{ torr})(303 \text{ K})} = 5.53 \text{ L}$$

2. WANTED: ? molecules gas
mL gas at STP

DATA: 6.02 x 10²³ molecules = 1 mole ("molecules" calls the Avogadro prompt)

1 mol any gas = 22.4 L any gas at STP (STP Prompt)

Strategy: Molecules and mL are wanted. The conversions use molecules, moles, and liters. The wanted is a ratio; all the data is in equalities. Try conversions.

SOLVE:

$$? \frac{\text{molecules gas}}{\text{mL gas at STP}} = \frac{6.02 \times 10^{23} \text{ molecules}}{1 \text{ mole}} \cdot \frac{1 \text{ mole gas}}{22.4 \text{ L gas STP}} \cdot \frac{10^{-3} \text{ L}}{1 \text{ mL}} = \boxed{2.69 \times 10^{19} \frac{\text{molecules}}{\text{mL gas at STP}}}$$

3. WANTED: ? P -- problem uses kPa units, and wants P at end = P₂DATA: P = 112 kPa = P at start = P₁t = 25°C initial ; K = 25°C + 273 = 298 K = T₁t = 100.°C final K = 100.°C + 273 = 373 K = T₂Strategy: There are 2 P's and 2 t's, but no V's. However, for a change in a sealed glass bottle, V₁ will equal V₂.

SOLVE:

$$\frac{P_1 \cancel{V_1}}{T_1} = \frac{P_2 \cancel{V_2}}{T_2}; \quad \boxed{\frac{P_1}{T_1} = \frac{P_2}{T_2}}; \quad ? = P_2 = \frac{P_1 T_2}{T_1} = \frac{112 \text{ kPa} \cdot 373 \text{ K}}{298 \text{ K}} = 140. \text{ kPa}$$

4. WANTED: ? $\frac{\text{g}}{\text{mol}}$ **MM**DATA: **m** 0.0147 grams gas = 1 mL gas NOT at STP **V** **mass/Vol** is **Density**20 °C = 20 + 273 = 293 K **T****P** = 1.00 atm

Strategy: If the gas were at STP, we could use conversions, but it is not. Try adding symbols.

The D and MM symbols fit the equation
$$P = \frac{D_{\text{grams}} \cdot RT}{MM}$$

You need an R value to use this equation. Since P is in atm, choose

R = 0.0821 atm • L / mol • K

SOLVE:

$$? = \frac{D_g}{\text{mol}} = MM = \frac{D_g}{P} \cdot RT = 0.0147 \frac{\text{g}}{\text{mL}} \cdot \frac{1}{1 \text{ atm}} \cdot 0.0821 \frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}} \cdot 293 \text{ K} = ???$$

Oops! The volume units don't cancel. I must have done something wrong. Hmm. My R uses liters, but the density was in mL. Let's add a conversion so that L and mL cancel.

$$? \frac{\text{g}}{\text{mol}} = MM = \frac{\text{density}}{P} \cdot RT = 0.0147 \frac{\text{g}}{\text{mL}} \cdot \frac{1}{1 \text{ atm}} \cdot 0.0821 \frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}} \cdot 293 \text{ K} \cdot \frac{1 \text{ mL}}{10^{-3} \text{ L}} = 354 \frac{\text{g}}{\text{mol}}$$

(It is better to convert all the data to the *consistent* units needed to cancel your R *before* you plug into the equation. This would have meant converting the "1 mL" in the density to 0.001 exact liters, in order to match the unit of volume in the R you chose. But the above, though messy, works. If the units cancel properly, it's probably right.)5. WANTED: ? °C **t**

DATA: 70.4 g UF₆
 352.0 g UF₆ = 1 mol UF₆ (grams prompt)
 4.48 L gas **V**
 202.6 kPa **P**
 R = 8.31 kPa • L / mol • K

(P uses kPa)

The symbols are t, V, P, R. You are missing n, but you can solve for moles from the data. Use?

* * * * *

$$\boxed{PV = nRT} \quad n = ? \text{ mol UF}_6 = 70.4 \text{ g UF}_6 \cdot \frac{1 \text{ mol UF}_6}{352.0 \text{ g UF}_6} = 0.200 \text{ mol UF}_6$$

$$T = \frac{PV}{nR} = \frac{(202.6 \text{ kPa})(4.48 \text{ L})}{(0.200 \text{ mol})(8.31 \frac{\text{kPa} \cdot \text{L}}{\text{mol} \cdot \text{K}})} = \frac{(202.6 \text{ kPa})(4.48 \text{ L})}{(0.200 \text{ mol})(8.31 \text{ kPa} \cdot \text{L})} \cdot \frac{\text{mol} \cdot \text{K}}{\text{mol} \cdot \text{K}} = 546 \text{ K}$$

$$^{\circ}\text{C} = \text{K} - 273 = 546 - 273 = 273 \text{ }^{\circ}\text{C}$$

7. WANTED: ? $\frac{\text{mol HF}}{\text{L HF soln.}}$ **t**

DATA: 994 mL HF *gas* at **SP** and 30.°C (= 0.994 L HF *gas*) **V**
 250. mL HF *soln.* (= 0.250 L HF *soln.*)

(This problem has *two* volumes, one for the *gas* and one for the aqueous *solution*. Label your volume units as *gas* or *soln.* so that you do not use the *solution* volume in the *gas* equation.)

Strategy: First, analyze the answer unit.

On top: since V, P, and T are known, *moles* of gas can be found with PV=nRT.

For the bottom unit, mL, and therefore *liters* of the solution, is known.

Any R can be used, provided P and V are converted to the R **units** chosen.

SOLVE: Find moles. $\boxed{PV = nRT}$

DATA: P = 1 atm (you can use standard pressure in *any* unit, so long as the P unit matches R)

V = 0.994 L HF *gas*

n = ?

R = 0.0821 $\frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}}$ (Any R can be used, as long as the units are consistent)

T in K = 30.°C + 273 = 303 K

$$? = n = \frac{PV}{RT} = \frac{1}{R} \cdot \frac{PV}{T} = \frac{\text{mol} \cdot \text{K}}{(0.0821 \text{ atm} \cdot \text{L})} \cdot \frac{(1 \text{ atm})(0.994 \text{ L gas})}{(303 \text{ K})} = \boxed{0.03996 \text{ mol HF}}$$

Separate a complex unit in the denominator, and carry an extra *sf* until the final step.

Using the answer for that part as DATA to find the WANTED unit.

$$\text{WANTED: } ? \frac{\text{mol HF}}{\text{L HF soln.}} = \frac{0.03996 \text{ mol HF}}{0.250 \text{ L HF soln.}} = \boxed{\frac{0.160 \text{ mol HF}}{\text{L HF soln.}}} = [\text{HF}]$$

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Module 18 — Gas Labs, Gas Reactions

Lessons 18A and 18B address graphing of experimental data and writing conclusions for lab reports which include graphs of data. Those lessons use gas laws as examples, but they may also be of benefit at any time that you are asked to interpret graphs of experimental results in chemistry, physics, applied math, or engineering.

Lesson 18A: Charles' Law; Graphing Direct Proportions

Timing: Lesson 18A should be done when you are asked to graph experimental data, discuss the meaning of a direct proportion, or work with Charles' law for gases.

Prerequisites: Lessons 17A on gas fundamentals and 17D on the ideal gas law.

* * * * *

Charles' Law: A Direct Proportion

For the special case of gas measurements when volume and temperature are varied, but moles and pressure are held constant, the ideal gas law $PV = nRT$ can be re-written as

$$\frac{V}{T} = \frac{nR}{P} = \frac{(\text{constant moles})(\text{gas constant } R)}{(\text{constant Pressure})} = (\text{a new constant}) = c$$

Multiplying and dividing the above three constants results in a new constant, a number with units that we will term c . The above equation can then be written as

$$\boxed{\frac{V}{T} = c} \text{ for an ideal gas, when moles and pressure are held constant.}$$

This is **Charles' law**, discovered in the early 19th century by Jacques Charles, a French scientist and hot air balloonist.

In the above equation, V and T are *variables* and c is a *constant*. Charles' Law means that in a gas sample, if moles and pressure are held constant, the numeric value of the ratio V/T will stay the same if you change V or T .

A *second* way to express this relationship: as long as no particles of gas enter or escape, and pressure is held constant, if an *initial* set of conditions V_1 and T_1 is changed to a *new* set of conditions V_2 and T_2 , the V/T ratio will be the same under both conditions. In equation form:

$$\frac{nR}{P} = c = \boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}} \text{ so long as pressure and moles are held constant.}$$

In this second form of Charles' law, if any three of the four V and T measurements are known, the fourth can be found using algebra, without finding or knowing c .

A *third* way to write Charles' law is

$$\text{since } \frac{V}{T} = c, \text{ we can write } \boxed{V = cT}, \text{ where } c \text{ is a constant.}$$

Using Charles' law in the form $V = cT$, it is easy to see that

- if the absolute temperature (T) is doubled, since c is constant, V must double.
- If the gas volume is reduced by half, it can only be because T in kelvins has been reduced by half.

Any relationship that fits the general equation $A = cB$, where A and B are variables and c is a constant, is called a **direct proportion**. The c in the equation is termed a **proportionality constant**.

Since V and T can vary but c is constant, Charles' law $V = cT$ is a direct proportion.

This leads to a *fourth* way to represent Charles' law in equations: since $V = cT$ is true, $V \propto T$. The \propto is a symbol for "is proportional to." The equation $V \propto T$ is read, "V is directly proportional to T."

Directly proportional simply means if one of the two variables changes by any multiple (such as: is quadrupled, cut by 2/3, or raised by 40%), the other variable must change by the same multiple.

Using this fourth equation, Charles' law can be translated into words as, "when moles and pressure of a gas are held constant, volume is **directly proportional** to absolute temperature."

To summarize, Charles' law can be written in four equivalent ways:

Charles' law: $\frac{V}{T} = c$ or $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ or $V = cT$ or $V \propto T$ when P and n are constant.
--

With the help of a flashcard, these four ways of writing Charles' law should be committed to memory. The *form* of these four equations can be used to describe *any* relationship between two variables that is a direct proportion, and many direct proportions are encountered in science. If any *one* of the forms of an equation representing a direct proportion is true, all four will be true.

The Loss of Ideal Behavior

One implication of Charles' law is that as an ideal gas gets colder and colder, its volume becomes smaller and smaller. At absolute zero, the gas volume would be zero.

In reality, when taken to lower temperatures and/or higher pressures, at some point all gases condense into liquids or solids. A gas loses most of its volume when it condenses, but that lower volume then stays close to constant as the liquid or solid is further cooled or compressed.

In general, the ideal gas law best predicts gas behavior at high temperatures and pressures at or below standard pressure. As any gas approaches a temperature and pressure at which it condenses, it no longer obeys the ideal gas law or laws such as Charles' law that can be derived from the ideal gas law.

* * * * *

Practice A: Answer all of the following *without* a calculator.

1. Which variables must be held constant for Charles' law to predict gas behavior?
2. Write four equivalent ways of expressing Charles' law.
3. A sample of gas is contained in a large syringe: a glass cylinder with a tightly sealed but moveable piston. The pressure of the gas inside the syringe is equal to the atmospheric pressure outside the syringe, which is held constant.

If the gas volume in the syringe is 30. mL at 200. K,

- a. What would be the volume of the gas
 - i. at 400. K?
 - ii. at 300. K?
 - b. What must the temperature be if the volume of the gas is changed to
 - i. 15 mL?
 - ii. 90. mL?
4. A sample of trapped gas molecules in a glass container (a constant volume) is attached to a pressure gauge. Pressure measurements are taken at a variety of temperatures.
 - a. Rewrite $PV=nRT$ to conform to the conditions of this experiment, with the 2 *variable* terms on the left, and the *constants* grouped on the right.
 - b. Will the P versus T relationship be a direct proportion? Why or why not?
 - c. The following measurements are recorded:
 - 1) At 150. K, P = 74 kPa
 - 2) At 200. K, P = 101 kPa
 - 3) at 250. K, P = 125 kPaWithin experimental error, are these data consistent with a direct proportion? Why or why not?
 - d. What pressure would you predict at a temperature of 450. K?

Graphing Direct Proportions

Many relationships can be investigated by changing one variable and measuring a second variable, while holding all other variables constant. In science experiments, you are often asked to graph the results of those experiments, looking for a relationship between two variables that results in a plot that is an approximate *straight line*. The data can then be interpreted based on the equation: $y = mx + b$.

As noted above, whenever a relationship can be expressed by an equation in the form:

$A = (\text{a constant}) \cdot B$, where A and B can vary, then A and B are directly proportional. It is also true that a *graph* of A versus B will produce a straight line which, if extended, passes through the origin. Why?

The “slope-intercept formula” for a straight line on a graph is $y = mx + b$, where

- **m** is the constant **slope** of the graph (the rise over the run), and
- **b** is the **y-intercept**, the value of y at $x = 0$ (which is the value of y where the graphed line crosses the y -axis).

If the value of the y -intercept (**b**) is zero, $y = 0$ when $x = 0$, and the straight line will pass through the origin (0,0).

For a graph of data for two variables that fall on a straight line that passes through the origin,

- the equation for the line is $y = mx + 0$, which simplifies to $y = mx$. Since the slope of a line is constant, this equation matches the form of a direct proportion: $A = (\text{a constant}) \cdot B$.
- The variables plotted on the y and x axis are directly proportional, and the constant slope **m** is the proportionality constant of the direct proportion $y = mx$.

If the value of the y -intercept (**b**) of a straight line is *not* zero, the y versus x relationship is said to be **linear**, but it is *not* directly proportional.

To summarize,

Direct Proportion = Line Through Origin

When two variables that are *directly proportional* are graphed, the data falls on a straight line thru the origin.

When graphed data for two variables fall close to a straight line through the origin, the variables are directly proportional, and the relationship can be described by the equation $y\text{-axis variable} = (\text{constant slope of line})(x\text{-axis variable})$.

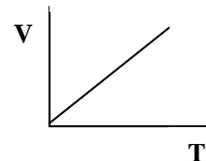
In science, a fundamental goal is to develop equations that measure how changes in one quantity will affect other quantities. Those equations can be written with relative ease when plots of experimental data fall on a straight line.

Graphing Charles' Law

Charles' law is a direct proportion. The form of Charles law $V = cT$ fits the general equation form: $y = (\text{a constant}) \text{ times } x$.

If in a Charles' law experiment, pressure and moles of a gas sample are held constant, the temperature of the gas is *varied* and recorded in kelvins, and the resulting volumes are recorded, the V versus T data can then be graphed.

Since $V = cT$ matches the form $y = mx + 0$, if V is plotted on the y -axis and T on the x -axis, the data falls on a line that looks like the graph at the right, a straight line through the origin. →



Since $V = cT$ matches the form $y = mx$, the value of the proportionality constant (**c**) will equal the slope (**m**) of the line through the origin.

* * * * *

Summary: Explaining the Graph of a Direct Proportion

If any *one* of the following statements is true, you may write that *all* of these statements are true.

1. When data for one variable Y is graphed on the *y*-axis, and data for the other variable X is graphed on the *x*-axis, the data plot (allowing for experimental error) as a straight line thru the origin.
2. A variable Y is directly proportional to a variable X.
3. $Y = m X$, where **m** is constant and is the slope of the line of Y plotted versus X.
4. $Y \propto X$ and $X \propto Y$
5. $\frac{Y_1}{X_1} = \frac{Y_2}{X_2} = c = m =$ the constant slope of a line fitting a graph of Y versus X.
6. For any measure of Y and X, the ratio Y over X will be constant.

Writing Lab Reports Based on Graphed Data

For a laboratory experiment, if a graph of data for two variables fit (allowing for experimental error) on a straight line that would pass through the origin, each of the above statements can be discussed as ways to express the relationship between the two variables.

Practice B: First learn the six rules in the summary above, then do these problems to test your knowledge. Check answers frequently.

1. A sample of trapped gas molecules in a container with a constant volume is attached to a pressure gauge. Measurements of pressure are taken at a variety of temperatures.
 - a. What are the symbols for the two gas variables in this experiment?
 - b. What are the symbols for the gas variables held constant in this experiment?
 - c. Starting from the ideal gas law, group all of the constants in this experiment into a single constant **c**, then write one equation with "**= c**" on the right and the variables of this experiment on the left.
 - d. Write four mathematically equivalent equations that express the relationship between P and T.
 - e. Express this relationship in words.
 - f. In this experiment, if pressure is plotted on the *y*-axis, and absolute temperature on the *x*-axis, what will be the shape of the graph?

2. For the graph at the right, label each of the following statements as **True** or **False**.

a. C and D are directly proportional.

b. $D \propto C$

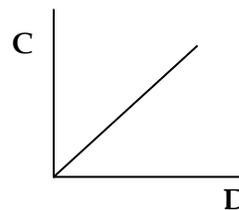
c. $D = mC$, (where m is slope)

d. $C = mD$

e. $D = (1/m) C$

f. $\frac{C_1}{D_1} = \frac{C_2}{D_2}$

g. $C_1D_2 = C_2D_1$



3. The equation for converting degrees Celsius to Fahrenheit is: $\text{°F} = (9/5) \text{°C} + 32$

Sample data points for this relationship are: $32 \text{°F} = 0\text{°C}$, $68 \text{°F} = 20 \text{°C}$, $212 \text{°F} = 100 \text{°C}$.

a. Will a graph of the data for °F on the y -axis, versus °C on the x -axis, fit on a straight line? Why or why not?

b. Is the relationship between Fahrenheit and Celsius a direct proportion?

c. For a plot of °F on the y -axis versus °C on the x -axis, what will be the value of the slope? What will be the value of the y -intercept?

ANSWERS

Practice A

1. In Charles' Law, *pressure (P)* and *particles of gas (n)* are held constant.

2. $\frac{V}{T} = c$ or $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ or $\boxed{V = cT}$ or $\boxed{V \propto T}$ when P and n are constant.

3a. i. at 400. K? **60. mL**. The absolute temperature doubled; the volume must double.

ii. at 300. K? **45 mL**. The Kelvin temperature rose 50%; the volume must rise 50%.

3b. i. 15 mL? **100. K**. If the volume is cut in half, the temperature in kelvins is halved.

ii. 90. mL? **600. K**. If the volume is tripled, the absolute temperature is tripled.

4. a. $\frac{P}{T} = \frac{nR}{V}$ (Variables on the left, constants on the right.)

b. **Yes**. The Part a equation can be written $\boxed{P/T = c}$. If either P or T doubles, the other variable must double.

c. **Yes**. The ratio P/T is about the same for all 3 points, which is one test for a direct proportion.

d. In each case the value of P in kPa is one-half T, so P would be predicted to be about $\frac{1}{2}$ of 450 = **~ 225 kPa**.

Practice B

1. a. **P and T** b. **V and n** c. Since $\frac{P}{T} = \frac{nR}{V}$ and n, R, and V are all constant, $\frac{P}{T} = c$.

d. $\frac{P}{T} = c$ and $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ and $P = cT$ and $P \propto T$ (when V and n are held constant.)

e. When volume and moles are held constant, gas pressure varies in direct proportion to absolute temperature.

f. A straight line through the origin.

2. a. C and D are directly proportional. **True**

b. $D \propto C$ **True** (if $C \propto D$, $D \propto C$)

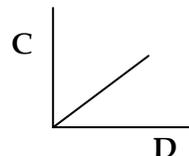
c. $D = mC$, (where m is slope) **False** ($x \neq my$)

d. $C = mD$ **True** ($y = mx$)

e. $D = (1/m)C$ **True**, since $C = mD$

f. $\frac{C_1}{D_1} = \frac{C_2}{D_2}$ **True**; since $C = mD$, $C/D = m = \text{constant}$

g. $C_1D_2 = C_2D_1$ **True**; since part (f) is true, and (f) and (g) are the same algebraically.



3a. Straight line? **Yes**, because $^{\circ}\text{F} = (9/5)^{\circ}\text{C} + 32$ is in the form $y = mx + b$. However, the line will not pass through the origin, since $b \neq 0$.

b. Direct proportion? **No**. A straight line represents a direct proportion only when the y-intercept is zero (the line passes through the origin). Here, the y-intercept is + 32. Another way to check: the y over x ratio would be constant for a direct proportion. For the data above, $^{\circ}\text{F}$ over $^{\circ}\text{C}$ is not constant.

c. Slope? Since $^{\circ}\text{F} = (9/5)^{\circ}\text{C} + 32$ is in the form $y = mx + b$, slope = $(9/5) = 1.80$

The y-intercept? $b = + 32$

* * * *

Lesson 18B: Boyle's Law; Graphs of Inverse Proportions

Prerequisites: Lessons 17A, 17D, and 18A.

* * * * *

Boyle's Law

When *moles* and *temperature* are held constant, the terms in the ideal gas law are

$$PV = nRT = (\text{constant moles})(\text{gas constant } R)(\text{constant } T)$$

Since multiplying constants results in a new constant, the above equation can be re-written as

$$\boxed{PV = c} \quad \text{where } c \text{ is constant when } n \text{ and } T \text{ are held constant.}$$

This is Boyle's law, discovered in the mid-1600's by the English scientist Robert Boyle. For any gas with ideal behavior, if moles and temperature do not change, the product *P times V* will stay the same as you change P and V.

Another way to express this relationship: if you have an initial set of conditions P_1 and V_1 , and you change to a new set of conditions P_2 and V_2 , the product $P \cdot V$ must stay the same, as long as no particles of gas enter or escape, and temperature is the same for the initial and final conditions.

$$\text{In equation form: } nRT = c = \boxed{P_1V_1 = P_2V_2} \quad \text{when } n \text{ and } T \text{ are held constant.}$$

If 3 out of the 4 variables among P_1 , V_1 , P_2 , and V_2 are known, the unknown 4th variable may be found using algebra (without using **R** or knowing **n**, **T**, or **c**).

From the form $PV = c$, it can be seen that if the pressure is *doubled*, the volume must drop by *half* for the product of P times V to remain constant. If the pressure is cut to one-third of an original value, it can only be because the original volume has tripled. This type of relationship is called an **inverse proportion**.

In words, for Boyle's law, "if the number of particles and the temperature of a gas are held constant, pressure will be **inversely proportional** to volume."

Since $PV = c$ can be re-written as $\boxed{P = c(1/V)}$, Boyle's law can also be written as

$$\boxed{P \propto 1/V}$$

This form is read as either "P is *inversely proportional* to V," or as "P is *directly* proportional to *one over* V." Recall that $1/V$ can also be written as V^{-1} .

One implication of Boyle's law is that as a gas is placed under higher and higher pressure, its volume will approach zero. In reality, under increasing pressure, at some point all gases lose their ideal behavior as they approach conditions at which they condense to liquids or solids, and the gas behavior will no longer be predicted by gas laws.

Summary: Commit to memory:

Boyle's law: $\boxed{PV = c}$ or $\boxed{P_1V_1 = P_2V_2}$ or $\boxed{P = c(1/V)}$ or $\boxed{P \propto 1/V}$ when gas **n** and **T** are held constant.

Practice A: Check your answers at the end of this lesson after each part.

- Write four equivalent mathematical equations expressing Boyle's law.
 - What two gas variables must be held constant for Boyle's law to predict gas behavior?
 - For the relationship $A \propto 1/B$,
 - If **A** is tripled, what must happen to **B**?
 - If **B** goes from 400 to 200, **A** must go from 300 to _____?
-

Graphing Inverse Proportions

When data for an inverse proportion such as $PV = c$ is graphed, the result is a portion of a **hyperbola**. Note that as x -values increase, y -values decrease. As x -values approach zero, y -values become large. This is the behavior of an *inverse* proportion.

$PV = c$ can be written as $P = c(1/V)$. If data for P is plotted on the y -axis and values calculated for $1/V$ are assigned to the x -axis, this form matches the general equation for a line $y = mx + b$ passing through the origin ($b=0$, see Lesson 18A).

A graph of the data for points representing of P and $1/V$ should therefore fall on a straight line through the origin, where the slope (m) of the line is the value of the constant c . The equation that explains and predicts values for P and V is $P = m(1/V)$.

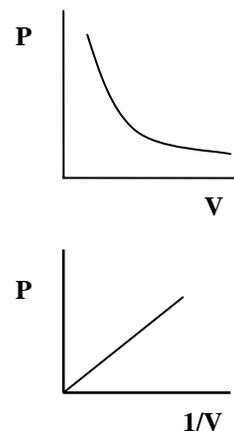
If the value of the y -intercept (b) is not zero, the y versus $1/x$ relationship will not be a direct proportion, and the data on the y - and x -axes will not be inversely proportional.

Summary: Explaining the Graph of an Inverse Proportion

If any one of these is true, all are true.

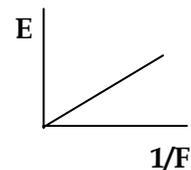
- Variable Y is inversely proportional to a variable X ;
- Variable Y is directly proportional to $1/X$;
- Y is equal to a constant times $1/X$;
- $Y = m(X^{-1})$, where m is the constant slope of a line on a graph of Y versus $1/X$
- $Y \propto 1/X$ (and $X \propto 1/Y$)
- $Y_1 X_1 = Y_2 X_2 = c = m =$ the constant slope of a line graphing Y versus $1/X$
- For any measure of Y and X , Y times X will be constant.
- When Y is graphed versus $1/X$, the points fall on a straight line thru the origin.

In general, for experiments in the laboratory in which the data results in a graph of Y versus X or $1/X$ with points that fall on a straight line thru the origin (allowing for uncertainty), discussion of the two variables as direct or inverse proportions, and equations that may be used to explain the data, may be appropriate in lab reports.



Practice B: Check your answers after each part or three.

1. For the graph at the right, label each of the following statements as True or False:
- E and F are inversely proportional.
 - $E = m(1/F)$ (where m is slope)
 - $F = m(E^{-1})$
 - $F = cE$ (where c is a constant)
 - $FE = (1/m)$
 - $FE = m$
 - $E_1F_1 = E_2F_2$
2. To cover a constant distance, the *rate* of travel (such as km per hour) is, by the definition of the terms, inversely proportional to the *time* required. Higher rate values will mean a smaller time values.
- Write the relationship between rate, and time, and a constant distance by completing these two equivalent equations.
(constant d) = _____ and rate = _____
 - Write the relationship between rate and time in two equivalent equations that use the α sign.
 - Express the equations above in words.
 - If the time for travel triples, the rate of travel _____.
 - To travel a fixed distance at various rates, if *rate* is plotted on the *y*-axis, and *time* on the *x*-axis, what will be the shape of the graph?
 - If rate is plotted on the *y*-axis, what must be plotted on the *x*-axis to produce a graph which is a straight line through the origin?

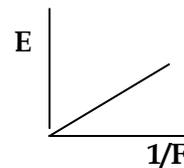
**ANSWERS****Practice A**

- Boyle's law: $PV = c$ or $P_1V_1 = P_2V_2$ or $P = c(1/V)$ or $P \propto V^{-1}$
- Two variables held constant for Boyle's law: T and n
- It may help to re-write $A \propto 1/B$ as $AB = c$
 - If A is tripled? B is reduced to 1/3 of its original value.
 - If B goes from 400 to 200, A must go from 300 to 600.

* * * * *

Practice B

1. a. E and F are inversely proportional. **True**
- b. $E = m(1/F)$ (m is slope) **True** This is a form of $y=mx+b$
- c. $F = m(E^{-1})$ **True.** If (b) is true, by algebra this is true.
- d. $F = cE$ (c is constant) **False.** This would be a direct proportion.
- e. $FE = (1/m)$ **False.** Part (b) is true, and therefore $1/m = 1/FE \neq FE$
- f. $FE = m$ **True.** This is equivalent to (b).
- g. $E_1F_1 = E_2F_2$ **True.** If FE is equal to a constant (part F), this will be true.
2. a. **Constant distance = (rate)(time) and rate = (constant d) / (time) or rate = (constant d) (1/time)**
- b. To use the proportional sign, start with a form $y = (\text{constant}) x$, which means $y \propto x$.
For this relationship, since **rate = (constant d) (1/time)**, then **rate \propto 1/time**;
and since this equation can be rewritten as **time = (constant d)(1/rate)**, then **time \propto 1/rate**.
- c. A few of several possibilities are, in traveling a constant distance, rate is inversely proportional to time; rate is directly proportional to one over time; constant distance equals rate times time.
- d. If the time for the travel triples, the rate of travel is **1/3 as fast**.
- e. Since rate and time are inversely proportional, the graph will be a section of a hyperbola.
- f. Since rate and time are inversely proportional, rate versus *1/time* fits on a line through the origin.



* * * * *

Lesson 18C: Avogadro's Hypothesis; Gas Stoichiometry

Timing: Do this lesson when you are assigned gas *reaction* calculations.

Prerequisites: Modules 2, 4, 5, 8, and 10, plus Lessons 11B, 17A, and 17D.

* * * * *

Avogadro's Hypothesis

In 1811, the Italian scientist Amedeo Avogadro made a remarkable discovery: that in chemical reactions, volumes of gases at the same temperature and pressure are used up and formed in simple whole-number ratios. To explain his results, he proposed that "equal volumes of gases, at the same temperature and pressure, contain equal numbers of molecules." Avogadro's hypothesis established the importance of counting particles in understanding chemical processes.

One implication of Avogadro's hypothesis is that for ideal gases, the chemical formula and molar mass of a gas have no effect on the volume that the gas will occupy.

- If samples of the same gas, different gases, or mixtures of multiple gases have the same pressure, temperature, and volume, they will contain the same number of gas molecules.
- For two samples of gas at the same temperature and pressure, if one sample has twice as many *molecules*, it will have twice the *volume*.

Gas Volumes and Coefficients

We have previously found that *coefficients* of a balanced equation can be read as ratios of

- *particles* (molecules, ions, or formula units);
- *moles* of particles (or any other multiple of particles); or
- *moles/liter* of particles that are all in the *same volume* (such as gas particles in a sealed glass container, or dissolved particles that react in the same aqueous solution).

In reaction equations, coefficients are ratios of particles, and gas volume ratios at the same temperature and pressure are also ratios of particles. Therefore, based on Avogadro's hypothesis we can add to the above list: *coefficients* of a balanced equation can also be read as

- *volumes of gases*, if the gases are measured at the same temperature and pressure.

Apply this rule to the following problem, and then check your answer below.

Q. Carbon monoxide oxidizes to form carbon dioxide. The balanced equation is



- Assuming that all of the gases are at the same temperature and pressure, starting with 12 volumes of CO;
- How many volumes of O₂ would be needed to completely use up the CO?
- How many volumes of CO₂ would be formed in the reaction?

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Answer

- The reaction *coefficients* are *ratios* of gas volumes for gases are at the same T and P. Twelve volumes of CO would require **six volumes of O₂** to be used up completely.
- Twelve volumes of CO plus six volumes of O₂ would be completely used up. Twelve volumes of CO₂ would be formed.

Practice A: Answers are at the end of this lesson.

- For the reaction $2 \text{H}_2(\text{gas}) + \text{O}_2(\text{g}) \rightarrow 2 \text{H}_2\text{O}(\text{g})$
 20. liters of hydrogen gas and 20. liters of oxygen gas, both at standard pressure and 120°C, are mixed and ignited. The resulting gases are adjusted to the original temperature and pressure. (At standard pressure and *above* 100.°C, all of the H₂O will be in the form of steam, a gas.)
 - Which is the limiting reactant?
 - For which of the reactants will some amount remain after the reaction?
 - After the reaction, which gases are present?

- d. How many liters of each of the reactants and products are present after the reaction is complete?

Gas Stoichiometry

All reaction calculations can be solved using the same fundamental stoichiometry steps. However, for gas reaction calculations, it will speed our work if we solve using these three variations in the stoichiometry steps.

- Method 1. IF the WANTED and *given* are both *volumes* of gases at the *same* temperature and pressure (even if not at *standard* T and P (STP)), *solve* based on Avogadro's hypothesis: by inspection or with one volume-volume conversion.
- Method 2. IF all of the gas volumes in the WANTED and *given* are at STP, *solve* using standard 7-step stoichiometry and the STP prompt.
- Method 3. IF the WANTED and *given* are not gases at the same T and P, or not all gases volumes in the DATA are at STP, *solve* using a rice table.

We will do one calculation for each type. A method 1 calculation was covered in the previous section. Let's try method 2.

Stoichiometry If All Gas Volumes Are At STP

In stoichiometry, if all gas *volumes* in the WANTED and DATA are at STP, you can solve using the STP prompt and the standard steps of conversion stoichiometry.

Try the following problem in your notebook.

- Q1.** The unbalanced equation for the burning of propane gas is



How many grams of O₂ gas are needed to burn 3.50 liters of propane at STP?

* * * * *

Answer

- WANT: ? g O₂ gas (Burning means reacting with O₂, so propane must be C₃H₈).
- DATA: 3.50 L C₃H₈ at STP (WANTED # *given* formula)
1 mol gas = 22.4 L any gas at STP (STP prompt)
32.0 g O₂ = 1 mol O₂ (g prompt)

For reactions, if WANTED formula ≠ *given* formula, use the stoichiometry steps.

Since the WANTED and DATA include only one gas volume and it is at STP, you can solve using conversions.

- Balance. $\text{C}_3\text{H}_8(\text{g}) + 5 \text{O}_2(\text{g}) \rightarrow 3 \text{CO}_2(\text{g}) + 4 \text{H}_2\text{O}(\text{g})$
- Bridge. 5 mol O₂ = 1 mol C₃H₈ (the WANTED to *given* mol ratio)

* * * * *

Steps 5-7: When a *single* unit is WANTED, convert “*final* unit WANTED = # and unit of *given* substance” to moles *given*, to moles WANTED, to units WANTED.

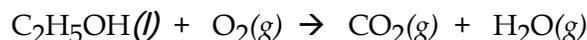
$$\begin{array}{ccccccc}
 \text{? unit WANTED} = & \text{unit given} & >> & \text{mol given} & >> & \text{bridge-mol WANTED} & > & \text{unit WANTED} \\
 \downarrow & & & \downarrow & & \downarrow & & \downarrow \\
 \text{? g O}_2 = & 3.50 \text{ L C}_3\text{H}_8 \text{ gas STP} & \bullet & \frac{1 \text{ mol gas}}{22.4 \text{ L gas STP}} & \bullet & \frac{5 \text{ mol O}_2}{1 \text{ mol C}_3\text{H}_8} & \bullet & \frac{32.0 \text{ g O}_2}{1 \text{ mol O}_2} = \boxed{25.0 \text{ g O}_2}
 \end{array}$$

To use conversions to solve gas stoichiometry, it is not necessary that *all* of the gases be at STP. The requirement is simply that *all* gas *volumes* in the WANTED and *given* be at STP.

In the problem above, the oxygen gas may or may not be at STP, but that does not affect the WANTED *mass* of oxygen. Since each substance has a constant molar mass, the relationship between the *moles* of gas and the *mass* of the gas will be the same at *any* P, V, and T.

Practice B

- The unbalanced equation for the burning of ethanol is



If the density of liquid ethanol is 0.789 g/mL, how many liters of CO₂ gas at STP will be produced by burning 2.50 mL of ethanol?

Stoichiometry If A Gas Volume Is Not At STP

The above two methods are for simple gas reaction situations in which either

- all reactants and products are gases at the *same* temperature and pressure, or
- all of the gases involved in the reaction are at *STP*.

For all other gas reaction calculations, our method to solve will be: use a *rice* table. A *rice* table takes more steps than some methods, but it has the advantage of using the same steps for every type of reaction amount calculation: whether solids or solutions, for gases whether at STP or not, in problems where you may or may not need to find the limiting reactant, and even for reactions that do not go to completion.

In Lesson 10G we learned a simple rule that can be used to decide whether to use conversions or a *rice* table to solve. That rule is:

To Solve Reaction Amount Calculations, Use Which Method?

If the reaction goes to completion and the *given* amount is clear, solve using conversion stoichiometry. In all other cases, or whenever you are not sure how to proceed, solve with a *rice-moles* table.

Another way to state this rule is: use a *rice* table for complex reaction calculations. Stoichiometry involving gases not at STP is complicated because to find moles of a gas *not* at STP, you need to use the ideal gas equation instead of conversions. Mixing equations

and conversions adds an element of complexity to the calculation, but the *rice* table will provide us with a consistent steps to keep track of data and solve.

The general rules for a *rice* table are: write the WANTED unit, then go from

All supplied units to all moles to *rice*-moles table to WANTED units

Specifically, in gas stoichiometry, if Avogadro's hypothesis cannot be used, and if the WANTED or DATA includes a gas *volume* not at STP, the steps are

- If conversions can be used to find moles, do so. If the *given* amount is a gas that is not at STP, use $PV=nRT$ to find the *moles* of the gas. Enter those moles into the *rice* table and solve the table.
- If the WANTED unit can be found by conversions from the WANTED moles in the table, do so.
- If the WANTED amount is a gas not at STP, solve the *rice* table, then use $PV=nRT$ to change moles of WANTED gas in the *End* row to the units of gas WANTED.

This method is best learned by example. Solve the following problem using the steps above. If you get stuck, read the answer until un-stuck, then try again.

Q2. A 0.972 gram sample of magnesium metal is reacted with an excess of hydrochloric acid to produce hydrogen gas. The unbalanced equation is



How many liters of H_2 would be formed at standard pressure and $20.^\circ\text{C}$?

($R = 0.0821 \text{ L}\cdot\text{atm}/\text{mol}\cdot\text{K}$)

★ ★ ★ ★ ★

Answer

1. **WANT:** ? L H_2 gas at SP and $20.^\circ\text{C}$

WANTED is a gas volume for a gas not at STP, and not all substances are gases at the same T and P, so we use the *rice* steps to solve:

All supplied units > all moles > *rice* moles table > WANTED units

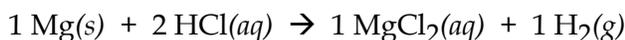
The only supplied substance data is g Mg. Convert to moles.

DATA: 0.972 g **Mg** (Start with a single unit)

1 mol Mg = 24.3 g Mg (grams prompt)

$$? \text{ mol Mg} = 0.972 \text{ g Mg} \cdot \frac{1 \text{ mol Mg}}{24.3 \text{ g Mg}} = 0.0400 \text{ mol Mg}$$

For a *rice* table, the first row needs a balanced equation.



Reaction	1 Mg	2 HCl	1 MgCl ₂	1 H ₂
Initial	0.0400 mol	excess	--	--
Change	- 0.0400 mol	- 0.0800 mol	+ 0.0400 mol	+ 0.0400 mol
End	0 mol	excess	+ 0.0400 mol	+ 0.0400 mol

In the *rice* table, in a reaction that goes to completion, one reactant must be limiting (totally used up). If there are two reactants, and one is in excess, the other will be limiting. The key row is the Change row, in which the ratios used up and formed must be the same as the reaction ratios (coefficients) in row 1.

Once the *rice* table is solved in moles, we can convert to other WANTED units.

WANTED: ? L H₂ gas at SP and 20.°C

DATA: 0.0400 mol H₂

To go from moles of a gas to liters not at STP, what equation is used?

* * * * *

$$PV = nRT$$

DATA: P = SP = 1 atm to match the P unit in the supplied R value

V = ? L H₂ gas = WANTED

n = 0.0400 mol H₂ (from the End row)

R = 0.0821 L·atm/mol·K (supplied)

T = 20.°C + 273 = 293 K

SOLVE:

$$? = V = \frac{nRT}{P} = \frac{(0.0400 \text{ mol})(0.0821 \text{ atm} \cdot \text{L})}{1 \text{ atm}} \cdot \frac{1}{293 \text{ K}} = 0.962 \text{ L H}_2 \text{ gas at SP and } 20^\circ\text{C}$$

Summary: Gas Stoichiometry

- IF the WANTED and *given* are both *volumes* of gases at the *same* temperature and pressure (even if not at *standard* T and P (STP)),
 - Solve by inspection or with one volume-volume conversion, based on Avogadro's hypothesis, using the coefficients of the balanced equation as whole-number gas *volume* ratios.
- If *all* of the *gas volumes* in the WANTED and *given* are at STP.
 - Solve using standard 7-step conversion stoichiometry and the STP prompt.
- If method 1 or 2 cannot be used, write the WANTED unit, then solve with a *rice* table and these steps:
 - All supplied units > all moles > *rice-moles* table > WANTED units

Practice C

1. In living cells, the sugar glucose is burned by this unbalanced equation.



How many liters of CO_2 , measured at 740. torr and $30.^\circ\text{C}$, can be formed by burning 0.250 moles of glucose? ($R = 62.4 \text{ torr}\cdot\text{L}/\text{mol}\cdot\text{K}$)

2. Sodium metal can be reacted with excess water to produce hydrogen gas. The unbalanced reaction equation is:



If 4.92 liters of H_2 gas is produced at 101 kPa and 25°C , how many grams of sodium reacted? ($R = 8.31 \text{ L}\cdot\text{kPa}/\text{mol}\cdot\text{K}$)

ANSWERS**Practice A**

1. a. H_2 b. O_2 . 20. L H_2 uses up only 10. L O_2 . c. O_2 and H_2O d. Zero H_2 , 10. L O_2 , 20. L H_2O .

Practice B

1. WANT: ? L CO_2 at STP
 2. DATA: 2.50 mL $\text{C}_2\text{H}_5\text{OH}$ (WANTED # *given* formula)
 1 mol gas = 22.4 L any gas at STP (STP prompt)
 0.789 g liquid $\text{C}_2\text{H}_5\text{OH} = 1 \text{ mL } \text{C}_2\text{H}_5\text{OH}$
 46.1 g $\text{C}_2\text{H}_5\text{OH} = 1 \text{ mol } \text{C}_2\text{H}_5\text{OH}$ (g prompt from density)

* * * * *

For reactions, if WANTED formula \neq *given* formula, use the stoichiometry steps.

Since the WANTED and DATA include only one gas volume, and it is at STP, you can solve using conversions.



4. Bridge. 2 mol $\text{CO}_2 = 1 \text{ mol } \text{C}_2\text{H}_5\text{OH}$ (the WANTED to *given* mol ratio)

* * * * *

Steps 5-7:

$$\begin{aligned} &? \text{ unit WANTED} = \text{ unit } \textit{given} \quad \gg \text{ mol } \textit{given} \quad \gg \text{ bridge} \quad > \text{ unit WANTED} \\ &\quad \downarrow \\ ? \text{ L } \text{CO}_2 \text{ STP} &= 2.50 \text{ mL } \text{C}_2\text{H}_5\text{OH} \bullet \frac{0.789 \text{ g } \text{C}_2\text{H}_5\text{OH}}{1 \text{ mL } \text{C}_2\text{H}_5\text{OH}} \bullet \frac{1 \text{ mol } \text{C}_2\text{H}_5\text{OH}}{46.1 \text{ g } \text{C}_2\text{H}_5\text{OH}} \bullet \frac{2 \text{ mol } \text{CO}_2}{1 \text{ mol } \text{C}_2\text{H}_5\text{OH}} \bullet \frac{22.4 \text{ L } \text{CO}_2 \text{ STP}}{1 \text{ mol } \text{CO}_2} \\ &= \boxed{1.92 \text{ L } \text{CO}_2 \text{ STP}} \end{aligned}$$

Practice C

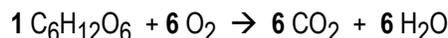
1. 1. WANTED: ? L CO_2 gas at 740. torr and 30°C (Want L, a single unit)
 2. DATA: 0.250 mol $\text{C}_6\text{H}_{12}\text{O}_6$ (This must be the single-unit *given*)

Burning is a reaction. For reactions, if WANTED \neq *given*, use stoichiometry steps.

Since WANTED is a gas volume for a gas not at STP, and not all substances are gases at the same T and P, use the *rice* steps to solve:

All supplied units > all moles > *rice-moles* table > WANTED units

The only supplied substance amount is moles glucose, so the “convert DATA to moles” step is done. For a *rice* table, the first row needs a balanced equation.



Reaction	1 $\text{C}_6\text{H}_{12}\text{O}_6$	6 O_2	6 CO_2	6 H_2O
Initial	0.250 mol	excess	--	--
Change	- 0.250 mol		+ 1.50 mol	
End	0 mol	excess	+ 1.50 mol	

“Burning” by definition means reacting something with excess O_2 . In a reaction that goes to completion, one reactant must be limiting (totally used up). If there are two reactants, and one is in excess, the other is limiting. The glucose is therefore the limiting reactant that determines all of the other amounts in the Change row..

Since an amount of CO_2 is the WANTED unit, we can leave out the other parts of the *rice* table.

- WANT: ? L CO_2 gas at 740. torr and 30°C
 DATA: 1.50 mol CO_2 gas at 740. torr and 30°C (from the **End** row)

To go from moles of a gas to liters not at STP, what equation is used?

* * * * *

$$\boxed{PV = nRT}$$

- DATA: P = 740. torr
 V = ? L H_2 gas = WANTED
 n = 1.50 mol CO_2
 R = 62.4 torr•L/mol•K
 T = $30^\circ\text{C} + 273 = 303 \text{ K}$

SOLVE:

$$? = V = \frac{nRT}{P} = \frac{(1.50 \text{ mol})(62.4 \text{ torr} \cdot \text{L})}{740. \text{ torr}} (303 \text{ K}) \cdot \frac{1}{1} = 38.3 \text{ L } \text{CO}_2 \text{ gas at } 740 \text{ torr and } 30^\circ\text{C}$$

2. 1. WANT: ? g Na
 2. DATA: 4.92 L H_2 gas at 101 kPa and 25°C
 23.0 g Na = 1 mol Na (g prompt)

For reactions, if WANTED \neq *given* formula, use stoichiometry steps.

Since the only supplied amount is a gas volume for a gas not at STP, and not all substances are gases at the same T and P, use the *rice* steps to solve:

All supplied units > all moles > *rice-moles* table > WANTED units

Job one is to convert all supplied units to moles.

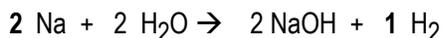
At the first step, since 25°C is not standard temperature, we cannot use the STP conversion to find moles. To convert L H₂ *given* to moles H₂ can be done with the supplied data and

$$\boxed{PV = nRT}$$

DATA: P = 101 kPa
 V = 4.92 L H₂
 n = ?
 R = 8.31 kPa•L/mole•K
 T = 25°C + 273 = 298 K

$$\text{SOLVE: } ? = n = \frac{PV}{RT} = \frac{(101 \text{ kPa})(4.92 \text{ L})}{(8.31 \frac{\text{kPa} \cdot \text{L}}{\text{mol} \cdot \text{K}})(298 \text{ K})} = \boxed{0.200 \text{ mol H}_2 \text{ gas given}}$$

For a *rice* table, the first row needs a balanced equation.



Solve this *rice* table backwards, from the known final moles of H₂.

* * * * *

Reaction	2 Na	2 H ₂ O	2 NaOH	1 H ₂
Initial	0.400 mol	excess	0	0
Change	-0.400 mol	- 0.400 mol	+ 0.400 mol	+ 0.200 mol
End	0 mol	excess	++ 1.50 mol	+ 0.200 mol

Since the initial grams of Na are WANTED, convert from initial moles Na.

* * * * *

$$? \text{ g Na} = 0.400 \text{ mol Na} \bullet \frac{23.0 \text{ g Na}}{1 \text{ mol Na}} = \boxed{9.20 \text{ g Na}}$$

* * * * *

Lesson 18D: Dalton's Law of Partial Pressures

Timing: Do this lesson when you are assigned calculations which include *Dalton's law*, *partial pressure*, *vapor pressure*, or *gas mixtures*.

Prerequisite: Lessons 18A and 18C.

* * * * *

Vapor Pressure

Liquids and solids have a tendency to become gases. At the surface of a liquid or solid, the vibrating molecules can break free and become part of the vapor above the liquid or solid.

Vapor pressure increases with increasing temperature. At higher temperature, the particles in a liquid and solid move faster, and they break free from the surface more often. For the particles that enter the gas phase, the kinetic energy of their collisions with the container walls creates gas pressure.

The **vapor pressure** above a liquid or solid is a *characteristic* of a substance at a given temperature. If the gas that leaves the solid or liquid is contained so that it cannot escape, the pressure caused by the vapor is predictable. If the substance and its temperature is known, this vapor pressure can be looked up in tables of chemical data.

A sample of vapor pressures for *water* is listed in the table at the right.

°C	H ₂ O vapor pressure
20.°C	17.5 torr
25.°C	23.8 torr
30.°C	31.8 torr
100.°C	760 torr

Boiling

A liquid boils at the temperature at which its vapor pressure equals the *atmospheric* pressure above the liquid. A liquid can be boiled at different temperatures by increasing or decreasing the atmospheric pressure on the liquid.

The “boiling” of a liquid is not the same as “evaporating.” Evaporation is a surface phenomenon; measurable evaporation will occur from all liquids (and many solids) at any temperature. However, a liquid “boils” only when gas bubbles can form below the surface of a liquid, and not just at its edges.

Gas Mixtures and Partial Pressure

Mixtures of gases with ideal behavior obey

Dalton's Law of Partial Pressures

The total pressure of a mixture of gases is equal to the sum of the pressures that the gases would exert alone.

The pressure exerted by each gas in the mixture is called its **partial pressure**. The partial pressure is the pressure that a gas would exert in the given volume if it alone were present.

For calculations involving gas mixtures, Dalton's law is useful in these forms which should be committed to long-term memory.

Dalton's Law Equations

To find the partial pressure of a single gas in a mixture:

$$P_{\text{gas one}} = (\text{mole fraction gas one})P_{\text{total}} = (\text{volume fraction gas one})P_{\text{total}}$$

For the total mixture:

$$\begin{aligned} P_{\text{total}} &= P_{\text{gas one}} + P_{\text{gas two}} + \dots (= \text{Partial P gas one} + \text{Partial P gas two} + \dots) \\ &= (\text{mole fraction gas one})P_{\text{total}} + (\text{mole fraction gas two})P_{\text{total}} + \dots \\ &= (\text{volume fraction gas one})P_{\text{total}} + (\text{volume fraction gas two})P_{\text{total}} + \dots \end{aligned}$$

This string of equalities for the total mixture provides us with a number of relationships that can be used to solve problems.

The mole and volume *fractions* in the above equations will always be numbers less than one: decimals which *total 1.00* for all of the fractions in a gas mixture.

$$\text{Fraction} = \frac{\text{part}}{\text{total}} \qquad \text{Mole Fraction} = \frac{\text{moles of part}}{\text{moles of total}}$$

Equations 2 and 3 above are similar because, by Avogadro's Hypothesis, for gases at the same temperature and under the same pressure, their volumes are proportional to their number of particles.

Use the above equations to solve the following problem.

Q. A mixture of 18.0 g of He gas and 16.0 g of O₂ gas in a sealed glass bulb is at a pressure of 120. kPa. What is the partial pressure of the He in the mixture?

* * * * *

WANT: ? P_{He} in kPa

DATA: 18.0 g He

4.00 g He = 1 mole He (g prompt)

16.0 g O₂

32.0 g O₂ = 1 mole O₂

120. kPa (total pressure)

Strategy: There are 4 equations that involve partial and total pressure. Which one should you use to solve? You know that

$$P_{\text{total}} = \text{Partial Pressure He} + \text{Partial P O}_2 = P_{\text{He}} + P_{\text{O}_2} = 120. \text{ kPa}$$

Since that equation has one known and two unknowns, you will need to use one of the other equations to gain more information. Which one? Check the data.

* * * * *

Should you use the equation using *mole* fractions, or *volume* fractions? We don't know the volumes, but from grams and the formula, we can find moles.

$$? \text{ Moles He} = 18.0 \text{ g He} \cdot \frac{1 \text{ mole He}}{4.00 \text{ g He}} = 4.50 \text{ moles He}$$

$$? \text{ Moles O}_2 = 16.0 \text{ g O}_2 \cdot \frac{1 \text{ mole O}_2}{32.0 \text{ g O}_2} = 0.500 \text{ moles O}_2$$

Equation (2) uses mole fraction.

$$\text{Total moles} = 4.50 \text{ moles He} + 0.500 \text{ moles O}_2 = 5.00 \text{ moles total}$$

$$? = \text{Partial Pressure He} = (\text{mole fraction He})(P_{\text{total}}) = \frac{4.50 \text{ moles He}}{5.00 \text{ moles total}} \cdot 120. \text{ kPa} = \mathbf{108 \text{ kPa}}$$

Practice A

Memorize the forms of Dalton's law, then use the equations to solve these.

- By volume, air is a mixture of 78% N₂, 21% O₂, and 1% other gases. At standard pressure, what is the partial pressure of the nitrogen gas in air, in mm Hg?
- A mixture of Ne and Cl₂ gases at standard pressure has a total mass of 24.3 grams. The mass of the neon is 10.1 grams.
 - What is the mole fraction of the chlorine gas in the mixture?
 - What is the partial pressure of the chlorine gas (in kPa)?

Gas Collected Over Water

Dalton's law is often used when collecting bubbles of a gas over water or above an aqueous solution. The resulting gas is a mixture which includes water vapor. The partial pressure of water vapor at a known temperature can be looked up in a table. Dalton's law will then allow calculation of the pressure of a gas that is collected over water.

$$P_{\text{total over water}} = P_{\text{gas wanted}} + P_{\text{water vapor from table}}$$

The partial pressure is the pressure that a gas would exert in the given volume if it alone were present. This means that the WANTED partial pressure in the equation above can be used with the ideal gas law to calculate other variables for that gas.

Practice B

Use the table of H₂O vapor pressure in the lesson above to solve these problems.

- Approximately what would be the vapor pressure of water at 23°C?
- If the total pressure of the gas in a mixture of H₂ gas and water vapor is 748 torr at 27°C, what is the partial pressure of the H₂ gas?
- A sample of oxygen gas (O₂) is collected over water at 25°C. If the volume of the gas is 6.0 liters and the total pressure of the gas mixture is 762 torr, how many moles of O₂ are in the sample? (R = 62.4 torr·L/mole·K)

$$? \text{ mol Cl}_2 = 14.2 \text{ g Cl}_2 \cdot \frac{1 \text{ mol Cl}_2}{71.0 \text{ g Cl}_2} = 0.2000 \text{ mol Cl}_2$$

To find the bottom WANTED unit (total moles), the moles of Ne are needed.

$$? \text{ mol Ne} = 10.1 \text{ g Ne} \cdot \frac{1 \text{ mol Ne}}{20.2 \text{ g Ne}} = 0.5000 \text{ mol Ne}$$

$$\text{mol total} = 0.2000 \text{ mol Cl}_2 + 0.5000 \text{ mol Ne} = 0.7000 \text{ mol total}$$

$$\text{WANTED: mol fraction Cl}_2 = \frac{\text{mol Cl}_2}{\text{mol total}} = \frac{0.2000 \text{ mol Cl}_2}{0.7000 \text{ mol total}} = 0.2857 = 0.286 \frac{\text{mol Cl}_2}{\text{mol total}}$$

1b. WANT: P_{Cl_2}

DATA: The equation that uses partial pressure and volume fraction of a single gas is

To find the partial pressure of a single gas in a mixture:

$$P_{\text{gas one}} = (\text{mole fraction gas one})P_{\text{total}} = (\text{volume fraction gas one})P_{\text{total}}$$

List the DATA next to the symbols of the equation you need.

$$P_{\text{gas one}} = P_{\text{Cl}_2} = ?$$

mole fraction $\text{Cl}_2 = 0.286$ as the decimal equivalent fraction from *part a*.

$$P_{\text{total}} \text{ in kPa} = \text{Standard pressure in kPa} = 101 \text{ kPa}$$

$$\text{SOLVE: } P_{\text{Cl}_2} = \text{mol fraction Cl}_2 \cdot P_{\text{total}} = 0.286 \cdot 101 \text{ kPa} = 28.9 \text{ kPa}$$

Practice B

1. **About 21 torr**, estimated from the table.

$$2. P_{\text{total}} = P_{\text{gas one}} + P_{\text{gas two}}$$

$$P_{\text{total over water}} = P_{\text{gas wanted}} + P_{\text{water vapor from table at } 27^\circ\text{C}}$$

$$P_{\text{gas wanted}} = P_{\text{total over water}} - P_{\text{water vapor}} = 748 \text{ torr} - \text{about } 27 \text{ torr} = 721 \text{ torr}$$

3. WANT: ? moles O_2 (**n**)

DATA: "Over water" means a gas *mixture* is present. Use

$$P_{\text{total over water}} = P_{\text{gas wanted}} + P_{\text{water vapor from table}}$$

$$P_{\text{water vapor at } 25^\circ\text{C}} = 23.8 \text{ torr}$$

$$25^\circ\text{C } \mathbf{t} \rightarrow \mathbf{T} = 25^\circ\text{C} + 273 = 298 \text{ K}$$

$$6.0 \text{ liters } \text{O}_2 \quad \mathbf{V}$$

$$762 \text{ torr} = \text{total pressure. } P_{\text{total}}$$

$$\mathbf{R} = 62.4 \text{ torr}\cdot\text{L/mol}\cdot\text{K}$$

Strategy: We want *moles* of gas. R is given, so we likely need R. Which of the two equations using R should be used? Label the data; see which fits.

$$PV = nRT$$

But to use $PV=nRT$ to find moles O_2 , we must know the pressure of *only* the O_2 .

$$P_{\text{total over water}} = P_{\text{gas wanted}} + P_{\text{water vapor from table}}$$

$$P_{\text{water vapor at } 25^\circ\text{C}} = 23.8 \text{ torr}$$

$$P_{\text{gas wanted}} = P_{\text{total over water}} - P_{\text{water vapor}} = 762 \text{ torr} - 23.8 \text{ torr} = 738 \text{ torr for } O_2$$

This tells us the pressure the O_2 gas would exert in the given volume if only the O_2 were present. That is what we need to know to use $PV=nRT$.

$$\text{SOLVE: } ? = n = \frac{PV}{RT} = \frac{(738 \text{ torr})(6.0 \text{ L})}{(62.4 \frac{\text{torr} \cdot \text{L}}{\text{mole} \cdot \text{K}})(298 \text{ K})} = 0.24 \text{ moles } O_2 \text{ gas}$$



b. WANT: ? moles H_2 gas

DATA: 0.210 g Li

$$6.94 \text{ grams Li} = 1 \text{ mole Li}$$

Since WANTED formula \neq given formula, we need the steps of stoichiometry.

Since this part involves moles rather than a gas volume, try conversion stoichiometry rather than a *rice* table.

Bridge. Add to DATA the WANTED to given mole ratio: 1 mol H_2 = 2 mol Li

$$\text{SOLVE: } ? \text{ moles } H_2 \text{ gas} = 0.210 \text{ grams Li} \bullet \frac{1 \text{ mole Li}}{6.94 \text{ g Li}} \bullet \frac{1 \text{ moles } H_2}{2 \text{ moles Li}} = 0.0151 \text{ moles } H_2$$

c. What will be the partial pressure of the H_2 ?

$$P_{\text{total}} = P_{\text{gas one}} + P_{\text{gas two}}$$

$$P_{\text{total over water}} = P_{H_2} + P_{\text{water vapor from table at } 20^\circ\text{C}}$$

$$P_{H_2} = P_{\text{total over water}} - P_{\text{water vapor}} = 758 \text{ torr} - 17.5 \text{ torr} = 740. \text{ torr}$$

d. i. At $20.^\circ\text{C}$ and its partial pressure? ? $V = 0.370$ liters – the volume of the mixture.

ii. At $20.^\circ\text{C}$ and standard pressure (Do not use R).

WANTED: $V_{H_2} = L \text{ H}_2$ (since the volume data is in liters) at $20.^\circ\text{C}$ and 760 torr V_2

DATA: $P_{H_2} = 740. \text{ torr} = P_{\text{initial}} \quad P_1$

0.370 L is the volume at 740. Torr = initial Volume V_1

760 torr = Pressure at end. P_2

$$t_1 = 20.^\circ\text{C} = t_2$$

Strategy: Assign symbols, pick equation, then solve for WANTED symbol.

$$\text{SOLVE: } \boxed{\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}} = P_1 V_1 = P_2 V_2 ;$$

$$? = V_2 = \frac{P_1 V_1}{P_2} = \frac{(740. \text{ torr})(0.370 \text{ L})}{(760 \text{ torr})} = 0.360 \text{ L at } 20.^\circ\text{C and SP}$$

* * * * *

Summary: Gas Labs and Gas Reactions

- Direct Proportions.** If any *one* of these is true, *all* of these statements are true.
 - Variable Y is directly proportional to a variable X;
 - Y is equal to a constant times X;
 - $Y = mX$, where **m** is the constant slope of the line for Y versus X
 - $Y \propto X$ (and $X \propto Y$)
 - $\frac{Y_1}{X_1} = \frac{Y_2}{X_2} = c = m =$ the constant slope of a line graphing Y versus X
 - For any measure of Y and X, the ratio Y over X will be constant.
 - When Y is graphed versus X, the data fits on a straight line thru the origin.
- Inverse Proportions.** If any *one* of these is true, *all* of these statements are true.
 - Variable Y is inversely proportional to a variable X;
 - Variable Y is directly proportional to $1/X$;
 - Y is equal to a constant times $1/X$;
 - $Y = c(X^{-1})$, where c will be the slope of a line on a graph of Y versus $1/X$
 - $Y \propto 1/X$ (and $X \propto 1/Y$)
 - $Y_1 X_1 = Y_2 X_2 = c = m =$ the constant slope of a line graphing Y versus $1/X$
 - For any measure of Y and X, Y times X will be constant.
 - When Y is graphed versus $1/X$, the points fall on a straight line thru the origin.
- Avogadro's Hypothesis:** Equal volumes of gases, at the same temperature and pressure, contain equal numbers of molecules.
- Coefficients** can represent *particles*, *moles* of particles, *moles/liter* of particles that are all in the *same volume*, and *volumes of gases* measured at the same T and P.
- For gas stoichiometry,**
 - If WANTED and *given* are gas volumes at the same T and P, solve using coefficients.
 - If all gas volumes are at STP, do *conversions* using the STP prompt.
 - If the above two methods cannot be used, use $\boxed{PV = nRT}$ and a *rice*-moles table.
- Dalton's Law of Partial Pressures:** The total pressure of a mixture of gases is equal to the sum of the pressures that the gases would exert alone.

7. **Dalton's law equations**

To find the partial pressure of a single gas in a mixture:

$$P_{\text{one gas}} = (\text{mole fraction gas})P_{\text{total}} = (\text{volume fraction gas})P_{\text{total}}$$

For the total mixture:

$$\begin{aligned} P_{\text{total}} &= P_{\text{gas one}} + P_{\text{gas two}} \quad (= \text{Partial Pressure gas one} + \text{Partial P gas two}) + \dots \\ &= (\text{mole fraction gas one})P_{\text{total}} + (\text{mole fraction gas two})P_{\text{total}} + \dots \\ &= (\text{volume fraction gas one})P_{\text{total}} + (\text{volume fraction gas two})P_{\text{total}} + \dots \end{aligned}$$

#

NOTE on the Table of Elements.

The atomic masses in this Table of Elements use fewer significant figures than most similar tables in college textbooks. By “keeping the numbers simple,” it is hoped that you will use “mental arithmetic” to do easy numeric cancellations and simplifications before you use a calculator for arithmetic.

Many calculations in these lessons have been set up so that you should not need a calculator at all to solve, if you look for *easy cancellations* first.

After any use of a calculator, use mental arithmetic and simple cancellations to *estimate* the answer, in order to catch errors in calculator use.

#

The ELEMENTS –

The **third** column shows the atomic number:

The **protons** in the nucleus of the atom.

The **fourth** column is the molar mass, in **grams/mole**. For radioactive atoms, () is the molar mass of most stable isotope.

Actinium	Ac	89	(227)
Aluminum	Al	13	27.0
Americium	Am	95	(243)
Antimony	Sb	51	121.8
Argon	Ar	18	40.0
Arsenic	As	33	74.9
Astatine	At	84	(210)
Barium	Ba	56	137.3
Berkelium	Bk	97	(247)
Beryllium	Be	4	9.01
Bismuth	Bi	83	209.0
Boron	B	5	10.8
Bromine	Br	35	79.9
Cadmium	Cd	48	112.4
Calcium	Ca	20	40.1
Californium	Cf	98	(249)
Carbon	C	6	12.0
Cerium	Ce	58	140.1
Cesium	Cs	55	132.9
Chlorine	Cl	17	35.5
Chromium	Cr	24	52.0
Cobalt	Co	27	58.9
Copper	Cu	29	63.5
Curium	Cm	96	(247)
Dysprosium	Dy	66	162.5
Erbium	Er	68	167.3
Europium	Eu	63	152.0
Fermium	Fm	100	(253)
Fluorine	F	9	19.0
Francium	Fr	87	(223)
Gadolinium	Gd	64	157.3
Gallium	Ga	31	69.7
Germanium	Ge	32	72.6
Gold	Au	79	197.0
Hafnium	Hf	72	178.5
Helium	He	2	4.00
Holmium	Ho	67	164.9
Hydrogen	H	1	1.008
Indium	In	49	114.8
Iodine	I	53	126.9
Iridium	Ir	77	192.2
Iron	Fe	26	55.8
Krypton	Kr	36	83.8
Lanthanum	La	57	138.9
Lawrencium	Lr	103	(257)
Lead	Pb	82	207.2
Lithium	Li	3	6.94
Lutetium	Lu	71	175.0
Magnesium	Mg	12	24.3

Manganese	Mn	25	54.9
Mendelevium	Md	101	(256)
Mercury	Hg	80	200.6
Molybdenum	Mo	42	95.9
Neodymium	Nd	60	144.2
Neon	Ne	10	20.2
Neptunium	Np	93	(237)
Nickel	Ni	28	58.7
Niobium	Nb	41	92.9
Nitrogen	N	7	14.0
Nobelium	No	102	(253)
Osmium	Os	76	190.2
Oxygen	O	8	16.0
Palladium	Pd	46	106.4
Phosphorus	P	15	31.0
Platinum	Pt	78	195.1
Plutonium	Pu	94	(242)
Polonium	Po	84	(209)
Potassium	K	19	39.1
Praseodymium	Pr	59	140.9
Promethium	Pm	61	(145)
Protactinium	Pa	91	(231)
Radium	Ra	88	(226)
Radon	Rn	86	(222)
Rhenium	Re	75	186.2
Rhodium	Rh	45	102.9
Rubidium	Rb	37	85.5
Ruthenium	Ru	44	101.1
Samarium	Sm	62	150.4
Scandium	Sc	21	45.0
Selenium	Se	34	79.0
Silicon	Si	14	28.1
Silver	Ag	47	107.9
Sodium	Na	11	23.0
Strontium	Sr	38	87.6
Sulfur	S	16	32.1
Tantalum	Ta	73	180.9
Technetium	Tc	43	(98)
Tellurium	Te	52	127.6
Terbium	Tb	65	158.9
Thallium	Tl	81	204.4
Thorium	Th	90	232.0
Thulium	Tm	69	168.9
Tin	Sn	50	118.7
Titanium	Ti	22	47.9
Tungsten	W	74	183.8
Uranium	U	92	238.0
Vanadium	V	23	50.9
Xenon	Xe	54	131.3
Ytterbium	Yb	70	173.0
Yttrium	Y	39	88.9
Zinc	Zn	30	65.4
Zirconium	Zr	40	91.2